Cyclic Stochastic Alternative Network Models for Project Management

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Abstract

Project implementation process modelling is the main active methodological body of the Project Management discipline [1]. The efficiency of decisions made and the whole functioning of the PM system is determined by the adequacy of models for real processes and their meeting the requirements of project management tasks and goals.

The high degree of complexity and laboriousness of drawing up timing schedules for numerous activities performed by many project members using a great range of resources, strict requirements for the quality of plans, the need for regular control of their fulfilment and adjustment call for the proper methods of solving problems of such sophisticated nature.

Today world market of Project Management Software[11] presents products with network models and methods of scheduling based on the researches of the end of 50th- beginning of 70th [2],[7] with very limited possibilities. At the same time the current mathematical methods of modelling project processes (classical network models [2], generalised [3, 4, 5], probabilistic [6] and stochastic [7] network models) do not always appear adequate to the complex reality of the modelled process. It should be noted that it refers to each method taken separately and to some combinations of these methods.

The paper describes a new class of network models adequately reflecting the complex project realisation process that are used for stating and solving optimal management tasks for this project. This class of models is a synthesis of generalized network models (with their rich spectrum of means for equivalence conversion of models [12,13] and describing the different logical and time interrelations between of project activities) with probabilistic and stochastic models to a considerable extent taking into account factors of risk and uncertainty the implementation of a project involves. These models (further referred to as cyclic alternative network models – CANMs) are the most flexible and adequate in the range of known tools for describing the process of managing and control over the development of a complex sophisticated project. CANMs offer all the advantages of generalised and stochastic models in comparison with traditional network models while at the same time involving just a slight complication of the language used for describing CANMs.

A general description of CANM category models was given in a number of works [8-10,14].

In the present paper a detailed mathematical description of CANMs is given and supplied with the substantiation of the requisite conditions of consistency as well as problem statements and CANM time analysis algorithms.

Keywords: Classical network models; Generalised network models; Probabilistic network models; Stochastic network models; Cyclic alternative network model

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1. Introduction

The paper describes a new class of network models adequately reflecting the complex project realisation process that are used for stating and solving optimal management tasks for this project. Each of the projects has a number of characteristics significant for the analysis using the methods, tools and means presented in this paper:

- the project consists of a certain set of interrelated activities the completion of which (all or a
- certain subset) means the completion of the project;
- the activities are partially ordered, i.e. must be implemented in a certain technological order;
- taking this order into account activities may start and finish independently of one another;
- some of the parameters of these activities are exposed to various random effects, they are, therefore, random in character;
- the technological order itself may very often depend on randomness and be of stochastic (alternative) nature.

Thus, we are considering the problem of project implementation process scheduling as a set of interrelated activities under the conditions of risk and uncertainty.

Besides, when stating and solving scheduling problems it is often necessary to take into account the scarcity of some resources or requirements for the dynamics of their consumption (for instance, the uniformity requirement). Moreover, some resources can be accumulated whereas it may be impossible to stock other resources in principle.

The high degree of complexity and laboriousness of drawing up timing schedules for numerous activities performed by many project members using a great range of resources, strict requirements for the quality of plans, the need for regular control of their fulfillment and adjustment call for the proper methods of solving problems of such sophisticated nature.

Project implementation process modeling is the main active methodological body of the Project Management tools [1]. The efficiency of decisions made and the whole functioning of the Project Management system is determined by the adequacy of models for real processes and their meeting the requirements of project management tasks and goals.

The current mathematical methods of modeling project processes (classical network models [2], generalized [3, 4, 5], probabilistic [6] and stochastic [7] network models) do not always appear adequate to the complex reality of the modeled process. It should be noted that it refers to each method taken separately and to some combinations of these methods.

The model presented in this paper for project management is a synthesis of generalized network models (with their rich spectrum of means for equivalence conversion of models [12,13] and for describing the logical structure of the set of project activities) with probabilistic and stochastic models to a considerable extent taking into account factors of risk and uncertainty the implementation of a project involves. These models (further referred to as *cyclic alternative network models – CANMs*) are the most flexible and adequate tool for describing the process of managing and control over the development of a complex sophisticated project. CANMs offer all the advantages of generalised and stochastic models in comparison with traditional network models while at the same time involving just a slight complication of the language used for describing CANMs. By that we mean the user's language of communication, means available to project managers (at different management levels) for describing projects, for participating in the interactive generation of timing schedules.

According to the three-dimensional classification of network models given in [10] CANMs fall under the most general category of cyclic stochastic alternative models.

Thus, according to [10] all known kinds of network models are a particular case of CANMs. In this connection the body of models and algorithms proposed herein can be taken as the basis of the development of a universal set of Project Management Software for multilevel network systems in project management with any degree of project complexity [8,9,12,13].

2. CANM description

The CANM is a finite oriented cyclic graph G (Ω,A) consisting of a set of events Ω and arcs (i, j) (i, j Ω) defined by the adjacency matrix $A=\{p_{ij}\}$. $0\leq p_{ij}\leq 1$, while $p_{ij}=1$ defines determinate arc (i, j), and $0< p_{ij} <1$ determines alternative event i which is connected with event j by an arc with probability p_{ij}. The set of arcs is divided into activity-arcs and link-arcs. The former denote a certain production output in the course of time, the latter exclusively reflect logical relationship between activities. An event can be the starting or the finishing time of activities fulfilled as well as some of their intermediate states.

Let T_i denote the time of event i, then the relation between the E of events connected with the arc (i, j) is assigned by an inequation:

$$T_j - T_i \geq \psi_{ij}, \qquad (1)$$

where ψ_{ij} in the general case is a random variable distributed according to a certain law within the interval from $-\infty$ to 0 or from 0 to $+\infty$.

Besides absolute constraints are possible to appear at the moment of event i occurrence:

$$l_i \leq T_i \leq L_i. \qquad (2)$$

Correlation (1) – (2) are the generalization of the appropriate inequations when generalized network models [3] where ψ_{ij} parameter and adjacency matrix A have a determinate pattern.

Let us consider the interpretation of correlation (1) provided the parameter ψ_{ij} is of a probabilistic nature.

If (i,j) is an activity-arc (or part of it) then the positively distributed random variable defines the distribution of the minimum duration of this activity (related to the maximum saturation of it with the determinant resource). Planning the maximal possible utilization of the resource for the activity we anticipate the fulfillment of the activity in the shortest possible time; contingencies and unforeseen complication and hindrances, however, condition the probabilistic character of this time, moreover, the mode (the most probable minimum time of the activity fulfillment) shifts to the right relative to the mathematical expectation as a rule. As a result of it the distribution of the variable ψ_{ij} is unimodal and asymmetric, and the type of distribution satisfying the requirements of the beta-distribution that was intuitively introduced for the estimation of activity duration in the PERT system first [7] and then was analytically and empirically validated and proved [6].

Thus, the minimum activity duration is a random variable $\psi_{ij}=t_{min}(i,j)$ distributed according top the law of beta-distribution in the interval [a,b] with probability density

$$\Phi(t)=C(t-a)^{p-1}(b-t)^{q-1}, \qquad (3)$$

In which C is determined such that $\int_a^b \varphi(t)dt=1$.

In [2] it is shown that parameters of ψ_{ij} distribution – $M\psi_{ij}$ and variance $\sigma^2\psi_{ij}$ - are approximately distributed according to the formulas:

$$M\psi_{ij}=(a_{ij}+4m_{ij}+b_{ij})/6, \qquad (4)$$

$$\sigma\psi_{ij}=(b_{ij}-a_{ij})/6, \qquad (5)$$

in which a_{ij}, b_{ij}, m_{ij} respectively are the optimistic, the pessimistic and the most probable estimates of the activity duration (i,j) prescribed by its executives (when using the three-estimates methodology). If the two-estimates methodology (proposed and substantiates in [6]) the probability density has the following form:

$$\varphi(t)=C(t-a)(b-t)^2, \qquad (6)$$

where $C=12/(b-a)^4$ and the distribution parameters

$$Mt=(3a+2b)/5, \qquad (7)$$

$$m=(2a+b)/3, \qquad (8)$$

$$Dt=0.04(b-a)^2. \qquad (9)$$

If the random variable ψ_{ij} in (1) corresponding to the activity-arc (i, j) is distributed in the interval from $-\infty$ to 0 then $-\psi_{ij} = t_{max}(j, i)$ assigns the distribution of the maximum duration of activity (j,i) (determined by the minimum saturation of it with the determinant resource). Applying to this variable the same procedure as the above described one we will obtain its distribution in the form (3) or (6) and the parameters calculated with (4)-(5) or (7)-(9) formulas respectively.

Assuming the most probable values (modes) as the values of these random variables we obtain in the particular case the known two-estimates probabilistic model (described in [6]) in which $a_{ij}=mt_{min}(i,j)$ and $b_{ij}=mt_{max}(i,j)$. Thus, the introduction of negatively distributed variables ψ_{ij} for activity-arcs (i,j) into (1) considerably extend possibilities of describing the time characteristics of activities which makes the widely used probabilistic model just one of particular cases.

For link-arcs (i,j) the variable ψ_{ij} assigns the distribution of the time dependence of events i and j, while the positively distributed variable ψ_{ij} determines the interconnection of "no sooner" kind (event j can occur no sooner than in ψ_{ij} days after the occurrence of event i) and the negatively distributed variable ψ_{ij} determines the interconnection of "no later" kind (event i can occur no later than $-\psi_{ij}$ days after the occurrence of event j).

In the latter case these links are called "reverse" [3].

In [3] describes in detail the wide opportunities available for setting technological links between activities using determinate parameters of ψ_{ij}; herein we have the generalization of these links which takes into account their possibly probabilistic character.

As the time of events T_i is calculated as the total duration of activities technologically preceding these events then if the number of these activities is rather large the distribution of the random variable T_i tends - according to the central limit theorem - to the normal distribution with the following parameters: mathematical expectation MT_i and variance DT_i. Normal distribution should be also anticipated for the parameter ψ_{ij} corresponding to the "reverse" arcs which is also proved by the statistical analysis [6].

Absolute constraints on the time of events assigned by (2) reflect the relevant directive, organizational and technological constraints on the times of accomplishing the activities or their elements set on the "absolute" (real or conventional) time scale. Absolute constraints are also characterized by the "no sooner" and "no later" type. The value of l_i and L_i are non-negative on the absolute scale. If we call the reference time (absolute or relative) a zero event then we can introduce arcs (0,i) and (i,0) with $\psi_{0,i}=1$ and $\psi_{i,0}=-L_i$ parameters respectively and (2) reduces to the form of:

$T_i - T_o \geq l_i,\ T_o - T_i \geq -L_i.$

Thus, absolute constraints of the (2) type are the particular case of constraints of (1) type for certain link-arcs.

Let us study now some additional opportunities for the description of the process of the complex and sophisticated project development that become available due to the introduction of a stochastic adjacency matrix A in combination with generalized links.

Let L(i,j) be a certain path linking the events i and j.

$$L(i,j)=\{i=i_o \rightarrow i_1 \rightarrow i_2 \rightarrow \ldots \rightarrow i_v=j\}. \qquad (10)$$

Let us call a path determinate if for all k [1,v] the following holds true: $pi^{k-1}i^k=1$, and stochastic if it does not. Thus, by definition the stochastic path contains at least one arc the probability of "occurrence" of which is rigorously smaller than 1. By the "occurrence " of an arc we mean here the completion of activity (for an activity-arc) and the fulfillment of requirements for the time connection of events (for link-arcs).

Let us define in the same way the determinate and stochastic loop $K(i)=\{i=i_o \rightarrow i_1 \rightarrow i_2 \rightarrow \ldots \rightarrow i_v=i\}$. (let us call such events i "loop").

Let the events i and j be linked by the path L(i,j), then the probability P(j/i) of the J event occurrence on condition that the event i has occurred is the production of the A adjacent matrix coefficients corresponding to the arcs of the linking path:

$$P(j/i)=\Pi^{v}_{k=1}\ pi^{k-1}i^k. \qquad (11)$$

If events i and j are linked by several paths an equivalent GERT-transformation of this network fragment is made in accordance with the formulas given in (7), generating function $\Psi_{ij}(s)$ is calculated and the probability of event j on condition that event i has occurred is $P(j/i)= \Psi_{ij}(0)$.

The first derived function $\Psi_{ij}(s)/\Psi_{ij}(0)$ with respect to s at point s=0 (the first moment $\mu_1(j/i)$) determines mathematical expectation M(j/i) of the event j time with respect to event i time. The second derived function $\Psi_{ij}(s)/\Psi_{ij}(0)$ with respect to s at point s=0 (the second moment $\mu_2(j/i)$) allows to calculate the variance of the event j time with respect to the event i time by the following formula:

$$\sigma^2(j/i)=\mu_2(j/i) - (\mu_1(j/i))^2. \qquad (12)$$

The GERT-transformation of a net fragment can be applied to the calculation of the probability of event j linked by stochastic paths with one node i to which a determinate full path leads. If stochastic paths from different alternative nodes i lead to event j in this case the following recurrent correlations are suggested:

$$P(j)=1 - \Pi_{\forall i<<j}(1 - P(i)p_{ij}), \qquad (13)$$

Where P(i) – the probability of event i , P(0)=1 and P(i) are already known for all i rigorously preceding j (∀i<<j). It follows from (13) that if event j is preceded by at least one full determinate path then P(j)=1 (one of the expressions between the brackets in the production becomes 0). We will call such event *determinate*, otherwise it is called *stochastic*.
Let us study a small numerical example illustrated in Fig. 1.

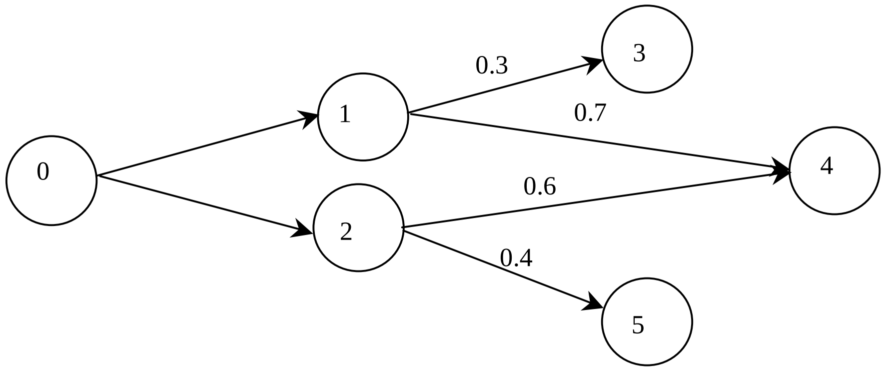

Fig.1. Example of stochastic model.

Here P(0)=P(1)=P(2)=1. GERT-transformation is not applicable to the calculation of P(4) (0.7+0.6=1/3>1). According to (13):

P(4)=1 – (1 – 0.7)(1 – 0.6)=0.88.

Below another simulation method of determining the probability of events is suggested. It is more effective for a larger network (number of arcs more than 300).

Path length L(i,j) is a random variable mathematical expectation ML(i,j) of which is a sum of mathematical expectations of the lengths of all arcs constituting this path, and variance DL(i,j) is equal to the sum of variances. Mathematical expectations of the lengths of arcs are calculated by formulas (4) or (7) and variances – by formulas (5) or (9) for three- or two-estimates methodologies respectively. Under these conditions the path (loop) length may take negative values which is interpreted in the following way (let us study an example of a determinate loop given in Fig.2):

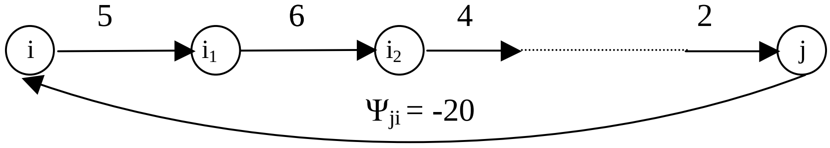

Fig.2 Example of a determinate loop.

In this case event j must occur no later than $-\psi_{ji}$ days after the occurrence of event i. As distinct from the generalized network models [3] parameter ψ_{ji} is probabilistic in character, which allows to describe logical-time relationship between events more flexibly

Statement of CANM time analysis problems

CANM time analysis problems as well as the time analysis of classical, generalized or stochastic network models form the basis of solving all the scheduling problems for project management. They are of particular significance themselves when dealing with project management without taking into account constraints on resources which is used for the creation of unique projects or projects of special importance.

Time analyses problems can also be separately used to generate different plan variants at certain values of the resources availability vector with the objective of their further comparison, plan variants quality evaluation and selection of ways and direction of its further improvement.

When solving any optimum scheduling problems CANMs time analysis algorithms are applied as an instrument for calculating the required parameters used in the relevant optimizations algorithms.

CANMs time analysis problems come to finding random vector $T=(T_0,T_1,\ldots,T_n)$ where T_i is the occurrence time of event i the coordinates of which satisfy inequations (1)-(2) and make certain efficiency function F(T) go to the extremum.

As $\{T_i\}$ here are random variables CANMs time analysis problems are characterized not only by the type of function F(T) but also the method of calculating $\{T_i\}$ and their parameters.

Owing to this let us single out the three categories of time analysis problems:
- *classical* which use mathematical expectations of al arcs lengths for $\{T_i\}$ calculation;
- *probabilistic* which calculate – on the basis of Liapunov central limit theorem or other mathematical tools - mathematical expectations $\{MT_i\}$ of events i times being the arguments of efficiency function F(T);
- *statistical* which determine for the given confidence level p on the basis of methodology [6] p-quantile estimates of empirical distribution of the times of events i – $\{W_p(T_i)\}$ – as well as values derivative of them and the values of efficiency function $F(W_p(T))$ where $W_p(T)=\{W_p(T_0), W_p(T_1),..,W_p(T_n)\}$

The form of efficiency function F(T) allows to calculate different types of plans (early, late, shortest possible and etc.) as well as a number of the required parameters (critical path, time reserves) for their further separate or auxiliary use

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4. CANM consistency concept

The cyclic alternative network model is called consistent if there is at least one feasible plan calculated for the relevant category of time analysis problems (classical, probabilistic or statistical) satisfying the set of ineqations (1)-(2).

Let us study these three concepts individually.

4.1.Classical model consistency

The mathematical expectations of all arcs lengths are calculated by the relevant formula (7), (4) (in the two- or three-estimates methodology) and assign a network with constant arc lengths. In the theory of classical network models [2] it is shown that the requisite condition of the model consistency is having no loops in it. Taking into account the stochastic character of the model under consideration and the fact that it has generalized links in it there may stochastic and determinate loops (cycles) in the CANM after doing the above-described calculations.

Lemma 1. For any confidential level α assigned in advance the presence of a stochastic loop does not result in the inconsistency of the model, namely we can state that the model will be consistent with probability α.

Proof.

Let loop K(i) and probability $P(i/i)<1$ of passing through it be assigned. The probability of leaving the loop for k-fold passage over it is calculated by the following formula: $1 – P^k(i/i)$. Taking this as the basis we calculate the number of possible times k of passing the loop after which we leave it with probability α: $\alpha=1 - P^k(i/i)$, therefore

$k=\ln(1-\alpha)/\ln P(i/i)$. (14)

For instance, for $\alpha=0.95$ and $P(i/i)=0/4$ we obtain $k\approx3$, i.e. after passing through the loop three times we will leave it with probability equal to 0/95. When determining (with probability α) a feasible time of event j identified with the leave the loop the length of the path going through event i up to event j should be summed up with kL(K(i)) where L(K(i)) is the length of loop K(i).

Lemma 2. In order to make the alternative model, for which arc lengths were calculated according to the classical procedure, consistent it is necessary and sufficient that the lengths of all the determinate loops (provided that there are no stochastic ones) are non-positive, i.e. $L(K(i))\leq0$ for all "loop" i.

Proof.

If the arc lengths are calculated according to the classical scheme and there are no stochastic loops we obtain a generalized network model for which the statement contained in lemma 2 is rigorously enough proved in [3].

Theorem 1. For the cyclic alternative model with arc lengths calculated according to the classical procedure to be consistent with a given probability α it is necessary and sufficient that the lengths of all the determinate loops are non-positive.

The proof of the theorem follows directly from the joint application of lemma 1 and lemma 2.

4.2.Probabilistic Consistency of the Model

We calculate mathematical expectation MT_i and variance σ^2T_i of event times using formulas from [6]. It should be noted that the values of parameters calculated by such analytical method are 15-20% different from those calculated by the classical method (on the basis of mathematical expectations of arc lengths).

We shall mean the *probabilistic inconsistency of the model on average* provided that the set obtained in the above-described way satisfies inequations (1)-(2) in which the mathematical expectation of ψ_{ij} is taken as its value.

Theorem 2. For the cyclic alternative model to be probabilistically consistent on average it is necessary and sufficient that the mathematical expectations of the lengths of all determinate loops are non-positive.

Proof.

Let K(i) be the loop and ML(K(i)) be the mathematical expectation of its length. Then the efficiency function of moments for K(i) loop is $M_{ii}(s)=e^{sML(K(i))}$. The first derived function $M_{ii}(s)$ with respect to s for s=0 (characterizing the mathematical expectation of the loop length) is an odd function with respect to the sign of the loop length. Function $\Psi_{ii}(s)=p_{ii}M_{ii}(s)$ is, therefore, odd in the same sense, p_{ii} being the probability of "entering" the loop and $P_b=1-$

p_{ii} being the probability of "leaving" it. As the efficiency function of the equivalent fragment is

$$\Psi_{ij}(s)= \Psi_b(s)/(1- \Psi_{ii}(s)), \qquad (15)$$

then for $p_{ii}<1$ we obtain:

$$p_{ij}=\Psi_{ij}(0)= \Psi_b(0)/(1- \Psi_{ii}(0))=(1-p_{ii})/(1- p_{ii})=1, \qquad (16)$$

i.e. we leave the stochastic loop with probability 1.
In order to determine the mathematical expectation of the equivalent fragment length let us calculate the first moment of (15) at the point s=0
:

$$M_1(j/i)=[p_b(1-p_{ii})ML(i,j)+p_b p_{ii} ML(K(i))]/(1-p_{ii})^2=$$

$$=ML(i,j)+ML(K(i))[p_{ii}/(1-p_{ii})]. \qquad (17)$$

Thus, in order to determine the average time of event j identified with leaving the loop it is necessary to add the length of the path going through event i to event j to $\delta L(K(i))$ where $L(K(i))$ is the length of loop K(i), and $\delta=[p_{ii}/(1-p_{ii})]$.

If the loop is determinate ($p_{ii}=1$) then for positive values of $\Psi_{ij}(s)$ and its derivative from (17) we can see the impossibility of leaving the loop (infinity of the equivalent fragment length). For non-positive loop length ML(K)i)) we have the probability of leaving it equal to 1 and the equivalent arc length lying within the interval from NL(i,j) to ML(i,j)+|ML(K(i))|. Proceeding from the assumption that T_i has normal distribution with the following parameters: mathematical expectation – MT_i and variance – $\sigma^2 T_i$ let us introduce a wider concept of *ε-probabilistic model consistency*.

Let us say that the CANM is ε-probabilistically consistent if there is ε>0 such that for all T_i satisfying the inequation $|T_i – MT_i|<\varepsilon$ correlations (1)-(2) hold true.

Theorem 3. For s cyclic alternative model to be ε-probabilistically consistent it is necessary and sufficient that the mathematical expectations of the lengths of all the determinate loops satisfy the following correlation: $ML(K(i))\leq-4\varepsilon$.

Proof.

Let K(i) be the loop and ML(K(i)) be the mathematical expectation of its length.

Let us single out the "positive path" and the "reverse" arc and without loss of generality do an equivalent GERT-transformation of this network fragment reducing to the form presented in Fig. 3.

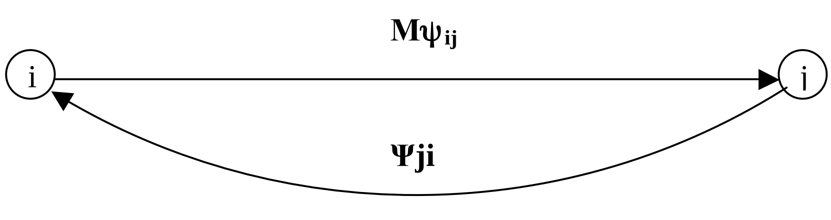

Fig. 3. Network fragment with the "positive" and the "reverse" arcs.

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Here $M\psi_{ij}$ is the mathematical expectation of the "positive" part of loop K(i).

Let correlation (1) hold true for T_i and T_j satisfying inequations:

$|T_i\text{-}MT_i|<\varepsilon, |T_j\text{-}MT_j|<\varepsilon,$ (18)

then they hold true for the extreme values of T_i and T_j minimizing the left part of (1), i.e. for

$T_i=MT_i$ $M\psi_{ij}\pm\varepsilon$ and $T_j=MT_j\pm\varepsilon$:

$MT_j\text{-}\varepsilon - (MT_i+\varepsilon)\geq M\psi_{ij}$ and $MT_i - \varepsilon - (MT_j+\varepsilon)\geq M\psi_{ji}$. (19)

Adding up the inequations we get the following
$-4\varepsilon \geq M\psi_{ij}+ M\psi_{ji} \approx ML(K(i))$ which proves the necessity of the statement of theorem 3.
In order to prove the sufficiency let $T_j=T_i+ M\psi_{ij}$. Inequation (1) for arc (i,j) remains valid. We have $T_i - T_j=- M\psi_{ij}$.
As $M\psi_{ij}+ M\psi_{ji}\approx ML(K(i))\leq-4\varepsilon$ then $- M\psi_{ij}\geq4\varepsilon+ M\psi_{ji}\geq M\psi_{ji}$. From which follows the validity of inequation (1) for arc (i,j).

Let us give a small numerical example illustrating the validity of theorem 3. Let $MT_i=50$, $MT_j=100$ for the network fragment presented in Fig.3. Let us assume $\varepsilon=1$ (one day). Correlation (1) for arc (i,j) must be valid for all T_i and T_j satisfying inequations (18) which means that it must be also valid for those T_i and T_j which minimize the left part of (1), i.e. for $T_i=50+1=51$ and $T_j=100 - 1=99$. It follows that the following inequation must hold true $M\psi_{ij}\leq T_j - T_i=99 - 51=48$. On the other hand, studying the "reverse" arc (j,i) and applying the same procedure we come to the validity of (1) for arc (j,i) for $T_i=50 - 1=49$ and $T_j=100+1=101$. Inequation $M\psi_{ji}\leq T_i - T_j =49 - 101= -52$ must, therefore, also be valid. Adding up these inequations we finally get the required correlation:

$M\psi_{ij}+ M\psi_{ji}\approx ML(K(i))\leq48 - 52= -4 = -4\varepsilon.$

The probabilistic consistency of the model on average is a particular case of ε-probabilistic consistency for $\varepsilon=0$.

4.3. Statistical consistency of the model

When using the statistical method of calculating the parameters of a network model we deal with p-quantile estimates of their values being the theoretical-probabilistic analogues of the relevant parameters [6]. Let us say that a cyclic stochastic model is *statistically consistent with probability p* if for each event i there are p-quantile estimates of events time $W_p(T_i)$ satisfying the following inequations:

$W_p(T_j) - W_p(T_i) \geq W_p(\psi_{ij}),$ (20)

$I_i\leq W_p(T_i)\leq L_i.$ (21)

Correlations (20)-(21) are probabilistic analogues of (1)-(2) here, and $W_p(\psi_{ij})$ is p-quantile estimate of the length of arc (i,j).
Theorem 4. For a cyclic alternative model to be statistically consistent with probability p it is necessary and sufficient that p-quantile estimates of the lengths of all the determinate loops satisfy correlation $W_p(L(K(i)))\leq0$
Proof.

After calculating p-quantile estimates the probabilistic model turns into a generalised network model, and the statement of theorem 4 is valid for it [3].

The existence of alternative nodes (with possible existence of stochastic loops) does not result in the inconsistency of the network according to lemma 1. The theorem, therefore, holds true for any CANM

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5.CANM time parameters calculation algorithms

5.1.Early and late time plans

We suggest the modified algorithm of "Pendulum" [3] for calculating early and late event time. The idea of the modification is to create a synthesis of the statistic method of calculating parameters applied for probabilistic networks [6] and the algorithm of "Pendulum" used in generalized networks [3-5] and to further apply it for CANMs (Fig. 4):

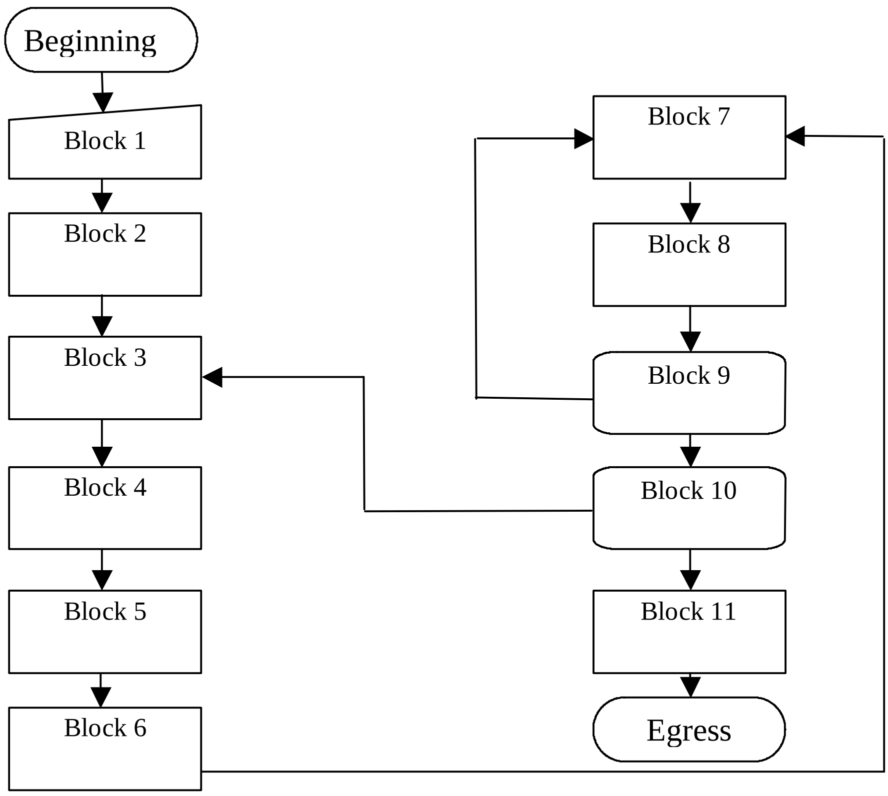

Fig. 4. Principle block diagram for calculating p-quantile estimates of early events time.

Block 1. Data input (matrix A coefficients, distribution parameters ψ_{ij}, confidence level). Network ordering. For small networks – calculations of P(j) according to GERT-transformation formulas and (13).
Block 2. Calculation of the required number of "drawings" N to ensure the given accuracy of results. The calculation made show that for p=0.95 and ε=0.05 we get N≈270.
Block 3. v:=v+1 (v – number of "drawing").
Block 4. The drawing of v variant of random variables ψ_{ij} – each in accordance with its law of distribution – obtaining constants $\psi_{ij}^{(v)}$ – the length of arc (i,j) for drawing v.

Block 5. The drawing of each alternative node i of going to the adjacent node j (discrete random variable p_{ij} represented by the line i of adjacent matrix A, $0< p_{ij} <1$ and $\sum_j p_{ij}=1$). The selected arc is marked and the others excluded from the graph. If in the resulting graph there appeared loop K(i) containing at least one marked arc it is a stochastic loop. Then we calculate its length $L^{(v)}K(i)$ and draw discrete variable P_{ij} again for node i. In accordance with lemma 1 one and the same stochastic loop for the given confidence level p can appear no more than k times where K is estimated with formula (14). The k-fold length of the loop is added to the length of the arc that was "drawn" at step (k+1) and go over to the analysis of the other stochastic loop (if there is one). In this process some inconsistencies (positive determinate loops) can appear in the network, then in accordance with (17) we add the δ-fold length on the loop, thus, estimating the time of the "leaving" event on average.

Block 6. The generalized determinate network $G^{(v)}$ we have obtained is divided into two networks $G_1^{(v)}$ and $G_2^{(v)}$ in such a way that neither $G_1^{(v)}$ nor $G_2^{(v)}$ contain any loops. The nodes of network $G_1^{(v)}$ are ordered by ranks in accordance with which the right numbering is set. Then this numbering is carried over to network $G_2^{(v)}$ and the initial $G^{(v)}$.

Block 7. For all nodes i of network $G_1^{(v)}$ we calculate the early time of

$$T_i^{0(v)}:\max_j\{T_i^{0(v)}, T_j^{0(v)} + \psi_{ij}^{(v)}\}.$$

Block 8. Then we do a sequence of manipulations similar to block 7 for the nodes of network $G_2^{(v)}$.

Block 9. If the results of blocks 7 and 8 do not coincide in at least one parameter we go back to block 7 (the number of these returns is not larger than the number of reverse arcs in $G_2^{(v)}$, otherwise – to block 10.

Block 10. If the number of drawing $v<N$ we go over to block 3, otherwise – to block 11.

Block 11. For each node i we calculate the number of its occurrences N(i). For determinate nodes N(i)=N, of course. P(i)=N(i)/N is a statistical characteristic of the probability of event i occurrence obtained by the method of simulation modeling. From the resulting population $\{T_i^{0(v)}\}$ for each node i we build up a variation series. Fix such a value of $T_i^{0(\zeta)}$ that $N_\zeta/N(i)=p$ where N_ζ is the number of terms of the variation series smaller than $T_i^{0(\zeta)}$. The value of $T_i^{0(\zeta)}$ is the sought-for quantile of the early time of event i - $W_p(T_i^0)$. In the same way we build p-quantile estimates of the arcs lengths $W_p(\psi_{ij})$ for variation series $\{\psi_{ij}^{(v)}\}$.

Variant v of generalized network model $G^{(v)}$ comes to the entrance of block 6 and, as a matter of fact, blocks 6 – 9 are a consolidated block diagram of algorithm "Pendulum" for the calculation of early event times in generalized network models. This algorithm is described in [3,4] in detail as well as the algorithm for calculating late event time. Applying this algorithm in blocks 7 and 8 we get $T_i^{l(v)}$ – late events time for the v-th variant of the generalized network model. Block 11 gives us $W_p(T_i^1)$ – p-quantile estimates of late events time.

5.2.Minimum duration plans

Duration of $L(T^{(v)})$ of any consistent $T^{(v)}=\{T_i^{(v)}\}$ of the variant v of network $G^{(v)}$ is defined by the formula:

$$L(T^{(v)})=\max_{ij}|T_i^{(v)} – T_j^{(v)}|. \qquad (21)$$

Replacing blocks 6 – 9 for the block of the test for a minimum of function (21) in the block diagram in Figure 4 we get the minimum duration plan for network $G^{(v)}$ (or a “compressed” plan). Value

$$L(T^{*(V)})=\min \max_{ij}|T_i^{(v)} - T_j^{(v)}| \qquad (22)$$

is the critical time of network $G^{(v)}$. A method of finding a compressed plan for a generalized network model is described in detail in [4] as well as the algorithms for building up four different kinds of compressed plans:

- early and late compressed plans for early completion of the project;
- early and late compressed plans for late completion of the project.

Using the method of finding a compressed plan for a generalized network model and getting the plans obtained as a result through block 11 we get the p-quantile estimates of the compressed plans.

5.3.Calculation of reserve, activity tightness coefficients, P-quantile estimates have critical, reserve and intermediate zones

Time floats for activity (i,j) corresponds here to their p-quantile analogues calculated by the formulas:

$$R^n_p(i,j)=W_p(T_j^n) - W_p(T_i^p) - W_p(\psi_{ij}) \text{ for a full reserve,} \qquad (23)$$

$$R^c_p(i,j)=W_p(T_j^p) - W_p(T_i^p) - W_p(\psi_{ij}) \text{ for a free reserve.} \qquad (24)$$

P-quantile coefficients of activity tightness are calculated by the following formula
:

$$W_p(k_H(i,j))=1- R^n_p(i,j)/ W_p(T_n^0) - W_p(T_{\kappa p}(i,j))), \qquad (25)$$

where $W_p(T_n^0)$ is the p-quantile estimate of the critical project implementation time, $W_p(T_{\kappa p}(i,j))$ is the p-quantile estimate of the duration of the coinciding with the critical path maximum path interval containing activity (i,j). $0\leq W_p(k_H(i,j))\leq 1$, moreover, the closer $W_p(k_H(i,j))$ is to 1, the relatively less time float has activity (i,j), the higher is, therefore, the risk of the failure to meet the set date of this activity.

Then the p-quantile critical zone, the p-quantile zone of reserves and the p-quantile intermediate zone [6] are determined:

- the p-quantile *critical zone* contains activities with $Wp\{K^H_{ij}\}>p_1$ where the value of p_1 is close to 1 ($p_1\approx 0.8\div 0.9$);
- the p-quantile *zone of reserves* comprises activities with values $Wp\{K^H_{ij}\}<p_2$, where p_2 is close to 0 ($p_2\approx 0.2$);
- the p-quantile *intermediate zone* $p_2\leq Wp\{K^H_{ij}\}\leq p_1$.

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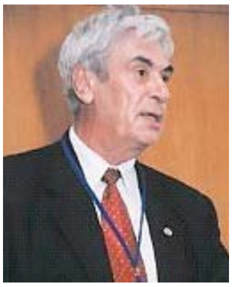	*Professor Vladimir Voropajev, Ph.D*  *Professor Vladimir Voropajev, PhD. is President and Chairman of the Board of the Russian Association of Project Management, SOVNET. Dr. Voropajev is professor of Project Management at the State University of Management, Moscow, Russia. He is also Head of the Program and Project Management Faculty for the Russian State Academy's Program for Professional Retraining and Professional Skill Development for Executives and Specialists in Investment Fields. He is a full member of the Russian Academy of Natural Sciences on Information Science and Cybernetics, and of the International Academy of Investments and Economy in Construction. From 1991 to 2001, he was Vice-president and a member of the Executive Committee of the International Project Management Association (IPMA), the global federation of national PM associations based in Zurich, Switzerland. He is an honorary Fellow of the Indian Project Management Association and a past member of the Global Project Management Forum Steering Committee. During his 40 years of engineering, scientific, teaching and consulting activities, he has published over 250 scientific research works including 7 monographs and 5 textbooks about the organization and planning of construction, information systems, and project management. Vladimir serves on the editorial boards of several international project management journals, is a frequent participant in PM conferences worldwide, and provides ongoing counsel and support to PM professional leaders in Azerbaijan, Kazakhstan, Ukraine, Yugoslavia and other countries. Professor Voropajev can be reached at voropaev@sovnet.ru.*

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