PMWorldToday

Published in *PM World Today* – June 2009 (Vol XI, Issue VI)

Principles of Top-Down Quantitative Analysis of Projects Part 2

Analytical Derivation of Functional Relationships between Project Parameters without Project Data

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Abstract

Modern quantitative project management is based entirely on the relationships obtained by the bottom-up processing of project data. Using these relations for the estimation and planning of projects leads to large errors and, ultimately, to the failure of many projects. This state of affairs motivates researchers to seek innovative ways to address these issues.

The purpose of this new direction of research is to find innovative methods of estimation and planning of projects, which, though based entirely on the daily experience of people and common sense, but does not directly depend on the specific project data.

This paper addresses the problems of estimation and planning of projects in an entirely new way. It is to develop a mathematical theory of projects without a specific project data. This new theory is based on the state equation of projects and the extremum principles that reflect the requirement of a minimum total project effort, minimum risk, etc.

As a result the differential equations of projects are derived and their analytical solutions are obtained.

Also relationships between project parameters are obtained in the same analytical way.

Introduction

Experience in theoretical physics and other sciences has shown that for a complete description of objects under investigation there is a need for both equation of state, and some extreme principles, that reflect their system-level behavior.

In this sense the classical example is the equilibrium static thermodynamics with its state equations and extremum principles, such as the minimum energy, maximum entropy and other principles. The rationale of this approach is that the equations of state describe all possible transitions of the system from one state to another, and the extremum principles are choosing one of these transitions only.

This allows establishing an unambiguous correspondence between the parameters of the system and deriving the functional relationships between them in an analytical way. This approach is widely accepted in modern science and its application to address the fundamental unsolved problems of contemporary project management is essential. The point here is that the contemporary quantitative PM is based on the data directly which is the main reason of project estimation large errors.

The top-down approach suggested in this work allows building a mathematical theory of project management which is data invariant and data independent. This new mathematical theory of projects potentially can replace the existing bottom-up unreliable statistical relationships between project parameters.

In all areas where the extremum principles are in use they are based on the notions of rationality of the corresponding processes. Similar principles of rationality are governing everyday human activities too. Such principles of rationality should also be an integral part of the description of the project works. This will help to build a theory of project management, similar to other quantitative sciences.

Both the equation of state and the principles of rationality are the generalizations of the experience in the specific field of knowledge. Therefore the results of the corresponding theories are generally in good agreement with the experiment.

Therefore in general these theories are data and experience based but also they are data independent because they are not based on the specific data directly.

Usually good theories are based not on the data directly but they are based on the generalizations of data. Classical mechanics, thermodynamics, classical electrodynamics, and other physical well known theories are the illustrations of this statement.

State equation of projects and extremum principles are exactly the generalizations of project data of this type. They are based on everyday human work experience and common sense.

The goal of this paper is to show that there are good possibilities to derive functional relationships between project parameters without using specific project data.

One of the goals of this paper is to show that this problem can be solved using a pure analytical top-down approach.

1. Change management and gradients of project parameters in project space

Change management needs to investigate the whole range of possible changes of project

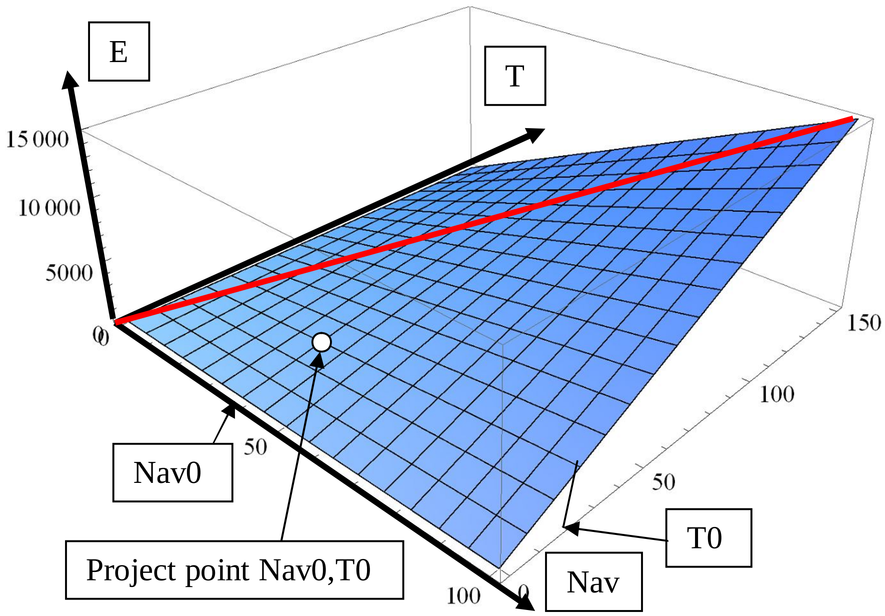


Fig.1 Project total effort surface with a project point in (T_0, N_{av0})

parameters from no change to maximum change. Therefore there is a need to investigate the gradients of project parameters which are the change directions of a maximum rate. Let's investigate the gradient of the total project effort E first.

Project total effort can be defined as the product of project duration T and project average staffing N_{av}

$$E = N_{av} * T \qquad (1)$$

Project space is presented in Fig.1. This is the project total effort surface with a project point in (T_0, N_{av0}). For change management it is necessary to investigate the directional derivative of the total effort and the direction of its maximum change. The point here is that any

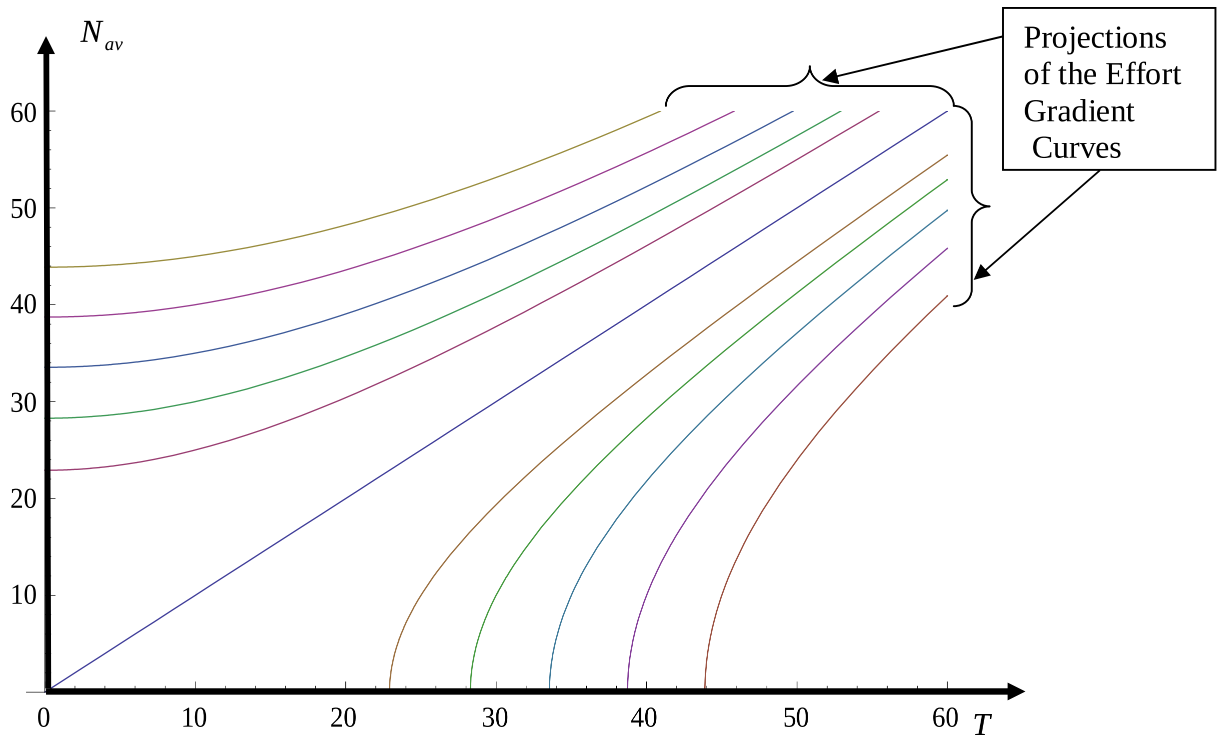


Fig.2 Projection of the effort gradient field on the project's duration and staffing (headcount) plane

change of project parameters can be between zero and maximum change of that parameter in the direction of the gradient. From (1) we can find the gradient of total effort as

$$\mathrm{Grad}E = \frac{\partial E}{\partial N_{av}} * \vec{i} + \frac{\partial E}{\partial T} * \vec{j} \qquad (2)$$

For partial derivatives from (1) we can have

$$\frac{\partial E}{\partial N_{av}} = T \text{ and } \frac{\partial E}{\partial T} = N_{av} \qquad (3)$$

Substituting (3) into the expression for gradient (2) we can have

$$\mathrm{Grad}E = T * \vec{i} + N_{av} * \vec{j} \qquad (4)$$

From here we can find [1] the parametric equation of the effort gradient curve in the project space as the solution of the following two differential equations:

$$\frac{dN_{av}}{dt} = T \quad , \qquad (5)$$

$$\frac{dT}{dt} = N_{av} \, . \qquad (6)$$

Eliminating parameter t from these equations we can obtain a new differential equation

$$\frac{dN_{av}}{dT} = \frac{T}{N_{av}} \quad . \qquad (7)$$

The solution of this new differential equation is the projection of the effort gradient curve in the space on the (N_{av}, T) plane. For the boundary condition (N_{av0}, T_0) the solution of differential equation (7) can be presented as

$$N_{av}^2 - N_{av0}^2 = T^2 - T_0^2 \qquad (8)$$

Solving this equation with regard to the average headcount one can obtain the relationship between the level of staffing and project duration for the effort gradient curve.

$$N_{av} = \sqrt{T^2 - T_0^2 + N_{av0}^2} \qquad (9)$$

This expression is presented in Fig.2 in the form of the family of curves.

The main conclusion from here is that the definition of project total effort (1) and the extremum requirement of GradE are able to generate a family of extreme trajectories or spatial curves that can govern the change management in projects.

2. Headcount gradient curve as the reflection of the minimum project effort principle

Requirement of the total effort minimum (minE) is a natural requirement for any project, since it is itself a direct consequence of the minimum expenditure requirement (minC) for the project development.

For the analysis of the requirement of minimum total effort let's consider two specific situations for an arbitrary project.

1. Assume we are somewhere in the middle of the execution of a project. Also assume due to some market requirement changes the requirement to the project duration is alleviated which means that project can be finished later. In its turn this means that project can be continued with smaller number of people.

2. Assume in the same situation the requirement to the project duration is strengthened and there is a need to finish it sooner. This means that project has to be continued with more people.

In these circumstances, the question arises about the possible reaction of the management. In the first case, assuming that people in average have the same productivity, it is necessary to remove the extra people as quick as possible in order to complete the remaining part of work with minimal effort.

In the second case it is necessary to add the needed number of people as quick as possible in order to minimize the relative amount of effort for completing the remaining part of work. As we can see, in both cases we needed to change the number of people as quick as possible having a goal to minimize effort.

This allows during top-down quantitative analysis of projects to replace the requirement of minimum effort (minE) with more straightforward and geometrically acceptable requirement of the headcount gradient GradN (meaning the gradient of the number of working people).

3. Differential equations of projects

Let's derive differential equations of an ordinary project using state equation of projects and the principle of headcount gradient.

State equation of projects is given by the expression [2]:

$$N_{av} = \frac{W}{P*T} \quad . \tag{10}$$

Changing any of the one of project parameters leads to unpredictable changes in the other parameters. In order to make them predictable let's use the principle of headcount gradient. From expression (10) we can find the gradient of N_{av} in the following form

$$\text{Grad}N_{av} = \frac{\partial N_{av}}{\partial W} * \vec{i} + \frac{\partial N_{av}}{\partial P} * \vec{j} + \frac{\partial N_{av}}{\partial T} \vec{k} \tag{11}$$

We can find partial derivatives $\frac{\partial N_{av}}{\partial W}$, $\frac{\partial N_{av}}{\partial T}$ and $\frac{\partial N_{av}}{\partial P}$ from (10):

$$\frac{\partial N_{av}}{\partial W} = \frac{1}{P*T}, \qquad (12)$$

$$\frac{\partial N_{av}}{\partial P} = -\frac{W_0}{P^2*T}, \qquad (13)$$

$$\frac{\partial N_{av}}{\partial T} = -\frac{W_0}{P*T^2}. \qquad (14)$$

Substituting expressions (12), (13) and (14) into the expression for gradient (11) we can have

$$\text{Grad}N_{av} = \frac{1}{PT}*\vec{i} - \frac{W}{P^2T}*\vec{j} - \frac{W}{PT^2}*\vec{k} \qquad (15)$$

This expression for the headcount gradient can serve as a basis for finding the parametric equation of the gradient curve in the project space [1]

$$\frac{dW}{dt} = \frac{1}{P*T}, \qquad (16)$$

$$\frac{dP}{dt} = -\frac{W_0}{P^2*T}, \qquad (17)$$

$$\frac{dT}{dt} = -\frac{W_0}{P*T^2}. \qquad (18)$$

In order to eliminate the parameter t it is necessary to divide equations (15), (16) and (17) in pairs. As a result we will have three new differential equations

$$\frac{dW}{dP} = -\frac{P}{W}, \qquad (19)$$

$$\frac{dW}{dT} = -\frac{T}{W}, \qquad (20)$$

$$\frac{dP}{dT} = \frac{T}{P}. \qquad (21)$$

Each of these equations can be obtained from the other two, that is independent of these are the only two.

Taking into account that the boundary conditions for the equations (19), (20) ¢ (21) are the coordinates of the project point in the project space W_0, N_{av0}, p_0 and T_0, their solutions will have the following form

$$W^2 - W_0^{\ 2} = -p^2 + p_0^{\ 2} \ , \tag{22}$$

$$W^2 - W_0^{\ 2} = -T^2 + T_0^{\ 2} \ , \tag{23}$$

$$p^2 - p_0^{\ 2} = T^2 - T_0^{\ 2} \ . \tag{24}$$

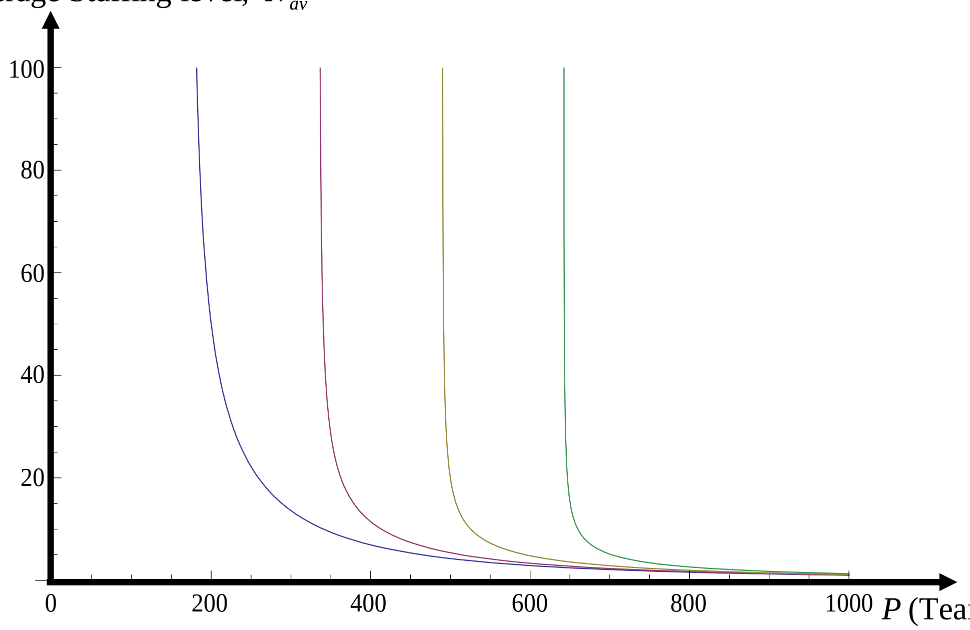
The joint solution of these equations allows finding "pure" functional relationships between the parameters of the project, having only two of the four current variables in each relationship.

Having functional relationships between project parameters (22), (23) and (24) and state equation of projects (10) it is possible to derive all other functional relationships between project parameters.

4. Derivation of functional relationship between Team Productivity and Team Size

This relationship has an important role in project scaling and correspondingly in project planning. It has been discussed repeatedly in the literature but still there are too many uncertainties associated with its interpretation. There are also a lot of misunderstandings related to the interpretation of project data on this subject.

Average Staffing level, N_{av}



P (Team Productivity)

Fig.3. Project average staffing level vs. team productivity for different constant values of project parameters

For the derivation of the functional relationship between team productivity and team size let's start with the state equation (10) from which for the work complexity we have

$$W = N_{av} * P * T . \qquad (25)$$

Substituting (25) into the equation (23) we will have

$$N_{av}^2 P^2 T^2 - W_0{}^2 = -P^2 + P_0{}^2 \qquad (26)$$

The next step is to eliminate the cycle time T from this equation. For that purpose we can find T from the equation (24)

$$T^2 = P^2 - P_0{}^2 + T_0{}^2 . \qquad (27)$$

Substituting this result into the equation (26) we obtain

$$N_{av}^2 P^2 .(P^2 - P_0{}^2 + T_0{}^2) - W_0{}^2 = -P^2 + P_0{}^2 \qquad (28)$$

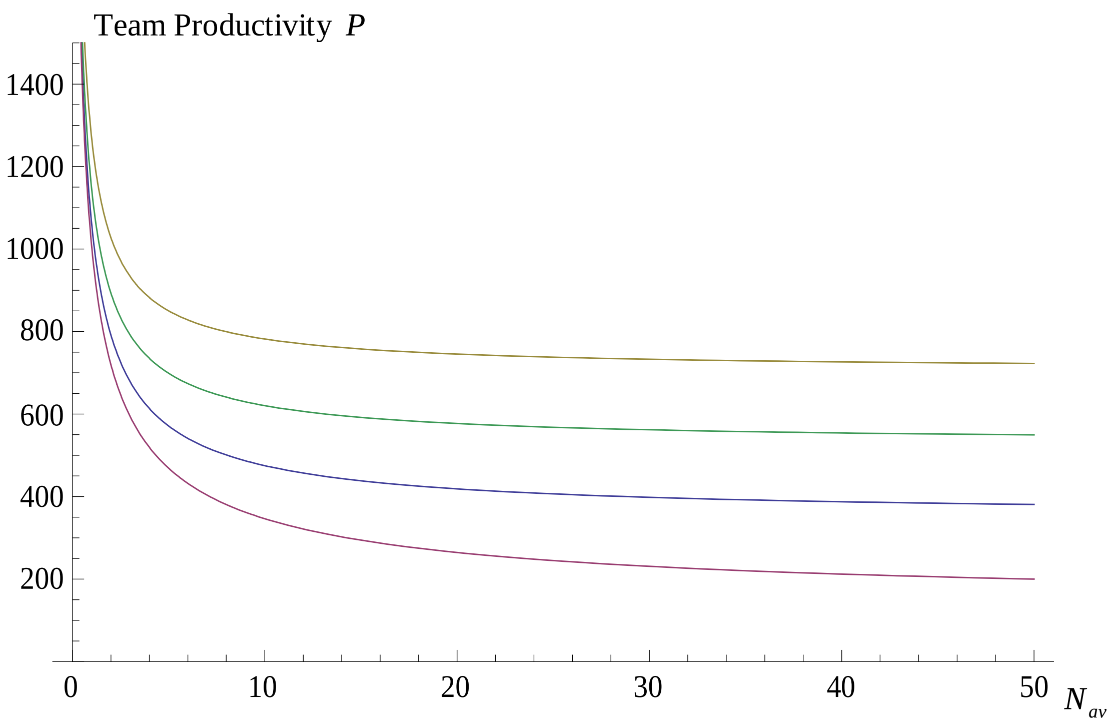


Fig.4 Team productivity vs. project average staffing level for different constant values of project parameters

This equation contains only two variables - N_{av} and P. Solving it with respect to the average number of people we can have

$$N_{av} = \frac{1}{P}\sqrt{\frac{W_0^{\;2} + P_0^{\;2} - P^2}{P^2 - P_0^{\;2} + T_0^{\;2}}} \qquad (29)$$

Fig.3 presents this relationship for different constant values of project parameters. Solving the same equation with respect to team productivity P we will have the following biquadrate equation

$$N_{av}^2 P^4 + [N_{av}^2 (T_0^2 - P_0^2) + 1]P^2 - W_0^2 - P_0^2 = 0 \qquad (30)$$

The solution of this algebraic equation is given by the following formulae

$$p = \frac{1}{\sqrt{2}} * \sqrt{(p_0^{\;2} - T_0^{\;2}) + \sqrt{(p_0^{\;2} - T_0^{\;2})^2 + \frac{4(W_0^{\;2} + p_0^{\;2})}{N_{av}^2}}} \qquad (31)$$

This is the well known functional relationship team productivity vs. team size. It is presented in Fig.4 in the form of the family of curves. The explanation of these curves is the following. Analysis of Fig.4 indicates that the relationship (31) can become an important starting point for a comprehensive understanding of project scaling rules.

In contrast to the prevailing views on the rapid decline in the productivity of people in large teams, based on a misinterpretation of project data, this result shows that the productivity of the team is slowly falling function of the number of people. This result is in agreement with the experience of people and just common sense. It is difficult to imagine that the work of the same person in a big team will be worse than his work in the relatively small team. The reason for that is that usually in both small and large teams people are working in small groups and their professional environment remains the same in the transition from small to large teams. Correspondingly their productivity change due to people interaction is small.

Also this relationship between team productivity and team size can help to interpret the project data correctly.

5. Relationship between average level of staffing and project duration for constant work complexity

The peculiarity of the consideration of the matter is that for the constant project complexity the total project effort is not a constant. As the planned duration of the project decreases its total effort increases markedly and this fact should be reflected in the derived formulas.

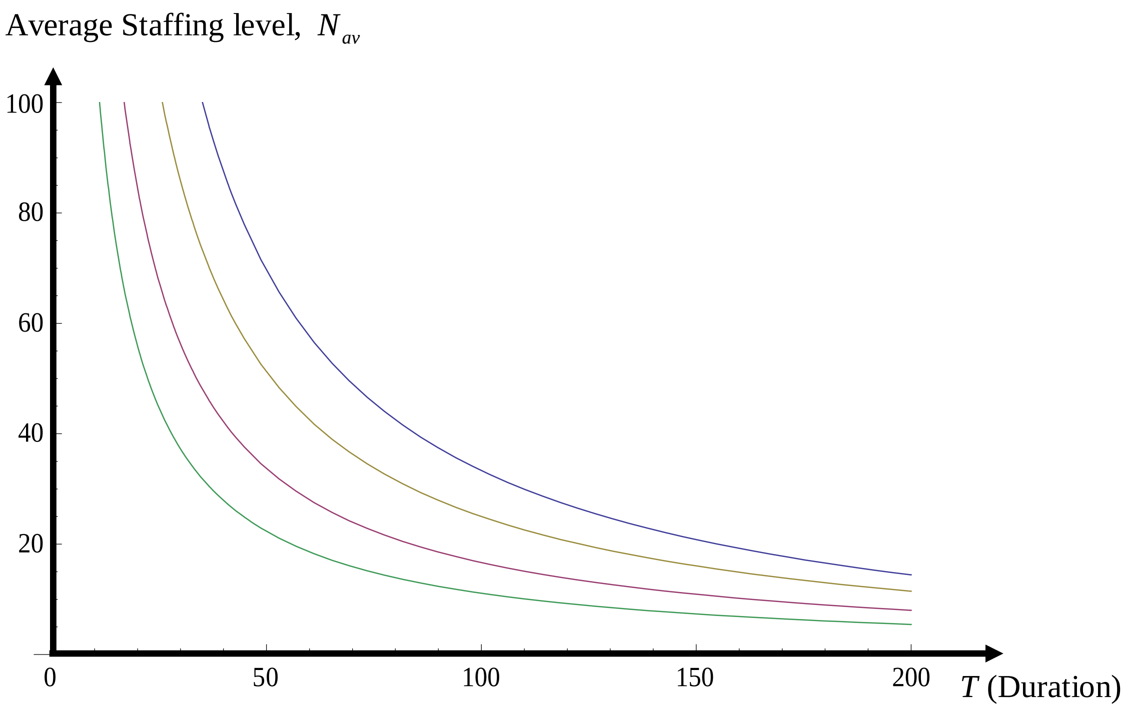


Fig.5 Team average staffing level vs. project duration for different constant values of project parameters

Let's start this derivation with expression (24). Solving this equation with respect to team productivity we can have

$$P^2 = T^2 - T_0^{\ 2} + P_0^{\ 2}. \qquad (32)$$

Substituting this result into the expression (26) we will obtain

$$N_{av}^2 . T^2 . (T^2 - T_0^{\ 2} + P_0^{\ 2}) - W_0^{\ 2} = -T^2 + T_0^{\ 2}. \qquad (33)$$

Solving this equation with respect to average staffing level N_{av} will result

$$N_{av} = \frac{1}{T} * \sqrt{\frac{W_0^{\ 2} + T_0^{\ 2} - T^2}{T^2 - T_0^{\ 2} + P_0^{\ 2}}} \ . \qquad (34)$$

This relationship is shown in Fig.5 for different combinations of constant values of project parameters.

Solving the same equation (33) with respect to the project duration one can obtain its dependence from the average staffing level for $W = W_0 = Const$.

$$T = \sqrt{\frac{N^2.(T_0^{\ 2} - P_0^{\ 2}) + 1 + \sqrt{(-P_0^{\ 2}.N^2 + T_0^{\ 2}.N^2 + 1)^2 + 4.N^2.(W_0^{\ 2} + T_0^{\ 2})}}{2.N^2}} \quad . \quad (35)$$

Taking into account that in $N^2(T_0^{\ 2} - P_0^{\ 2}) >> 1$ it is possible to simplify the formulae (35)

$$.T = \frac{1}{\sqrt{2}}.\sqrt{T_0^{\ 2} - P_0^{\ 2} + \sqrt{\frac{(T_0^{\ 2} - P_0^{\ 2})^2 + 4.(W_0^{\ 2} + T_0^{\ 2})}{N^2}}} \qquad (36)$$

6. Relationship between team productivity and project duration

The same basic equation (24) can serve as a basis for the derivation of the relationship between team productivity and project duration for constant work complexity.

Solving this equation with respect to the team productivity we can obtain the subject relationship.

$$P = \sqrt{T^2 - T_0^{\ 2} + P_0^{\ 2}} \qquad (37)$$

This relationship is presented in Fig.6 as a family of curves for different boundary conditions. Outwardly these curves are similar to the $N_{av}(T)$ curves in Fig.2, but this is only a geometrical similarity. The essence of these two families of curves is quite differing from each other.

Explanation of the Fig.6 is the following. For the constant work complexity value W_0 the growth of the project duration is accompanied with the decline of the project's average staffing level. In its turn the decreasing average staffing level leads to the team productivity growth due to the decreasing interaction between the team members.

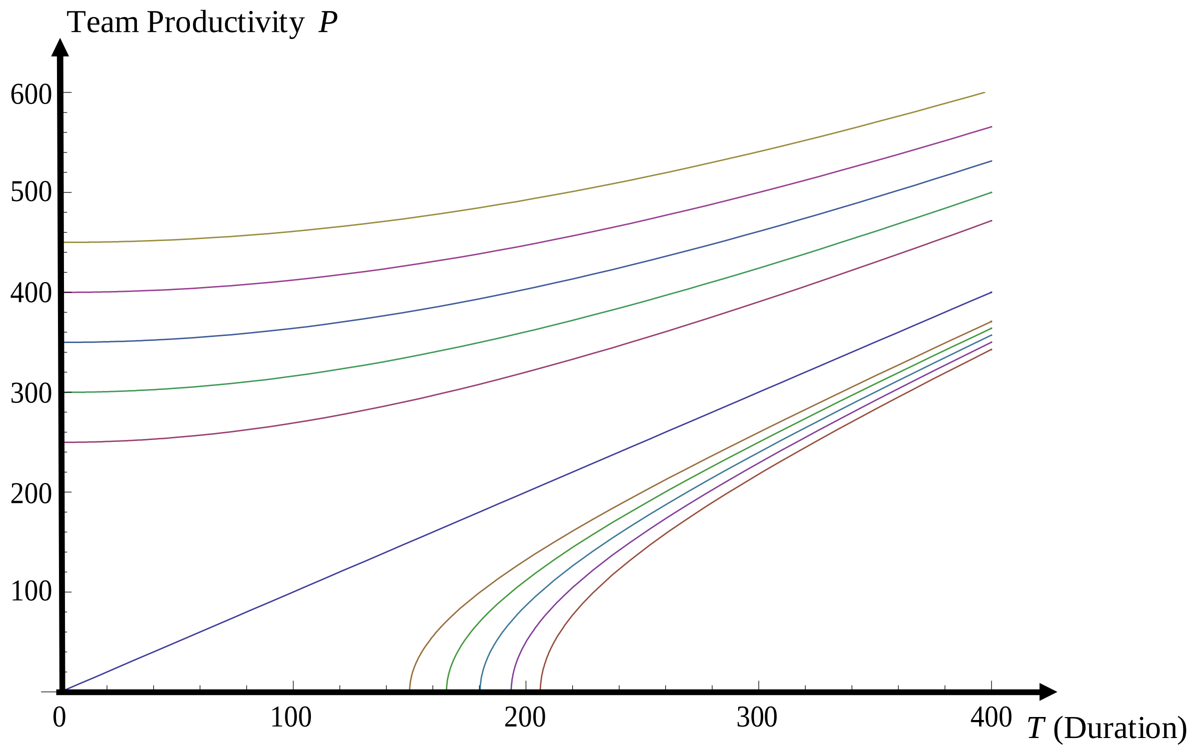


Fig.6 Relationship between team productivity and project cycle time

7. Relationship between project total effort and other parameters for constant project complexity

The derived analytical results provide an opportunity to obtain functional relationships between project effort and the other parameters too. It should be noted that if we have obtained, for example, the analytical relationship between project total effort and duration, then this dependence does not contain the variables of team productivity and staffing level in an explicit form, but takes into account their influence indirectly. That is the meaning of the analogy with the classical thermodynamics.

Relationship between total effort and project duration for constant project complexity

Taking into account that always $E = N_{av} * T$ form expression (34) we can have

$$E = N_{av} *.T = \sqrt{\frac{W_0^{\ 2} + T_0^{\ 2} - T^2}{T^2 - T_0^{\ 2} + p_0^{\ 2}}} \qquad (38)$$

This relationship is presented in Fig.7.

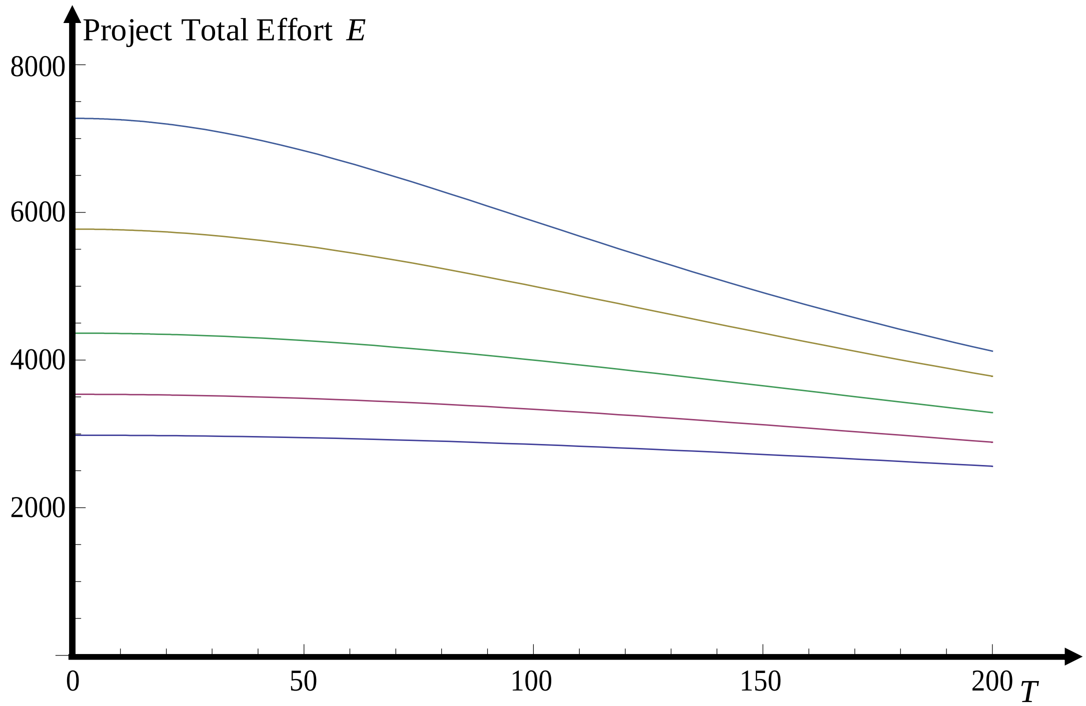


Fig.7 Project total effort vs. project duration for different constant values of project parameters

Analysis of these curves indicates:

a) The increase in the duration of the project leads to a reduction in project effort due to reducing the average number of people working on the project and corresponding increase in the productivity of their work.

b) The big project corresponds to a greater decline in the overall effort of the project because of the relatively large decrease of the number of working people.

c) The small duration of the project corresponds to a constant value of the effort because of the constancy of the productivity of the large number of people.

Qualitative analysis of the curves presented in this figure serves as a proof for the theoretical top-down approach to the problems of project management.

Relationship between total effort and average staffing level for constant project complexity

This relationship can be derived by substituting the expression (34) into the formula for the total effort.

$$E = \frac{1}{\sqrt{2}} . \sqrt{N^2.(T_0^{\ 2} - P_0^{\ 2}) + 1 + \sqrt{((-P_0^{\ 2}.N^2 + T_0^{\ 2}).N^2 + 1)^2 + 4.(W_0^{\ 2} + T_0^{\ 2})}} \quad (39)$$

This function is presented in Fig.8.

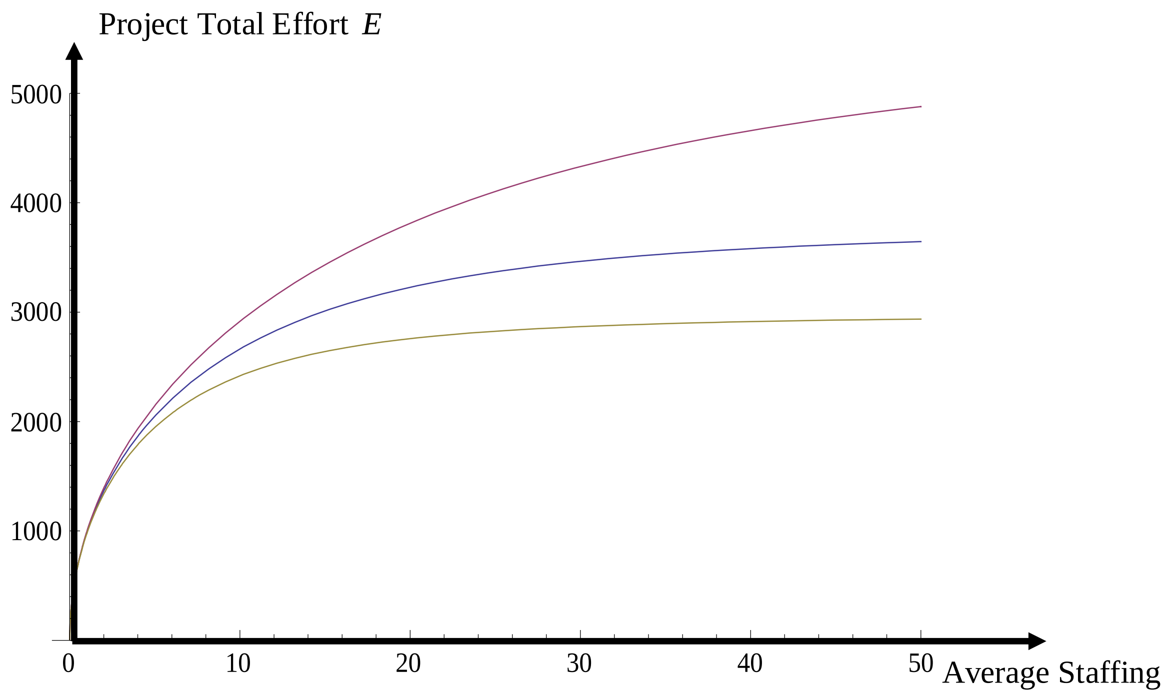


Fig.8 Project total effort vs. project average staffing for different constant values of project parameters

Main results

- Universal mathematical model of projects in the form of a system of differential equations is developed;
- Functional relationships between project parameters are derived;
- Top-down mathematical theory of projects without a direct usage of the specific project data is developed;
- Non-linear theory of project transformation is developed.

Conclusions

1. New mathematical theory of projects is applicable for an arbitrary project irrespective to the specific field of application.
2. This theory can serve as a basis for a new project data collection and data mining methodology. It opens new opportunities for project data structuring and interpretation.
3. All the conclusions of the new project theory are made on a uniform basis, so they do not have internal contradictions.
4. Project theory based conclusions also are not in a conflict with everyday human experience and common sense.
5. New project theory is capable to make various predictions and estimations that can be verified qualitatively and quantitatively.
6. Gradient principle is not the only possible way to build a mathematical theory of projects. There may be other approaches to this problem based on other extremum principles.

Future work is connected with:

1. Development of the theory of projects with variable project size and difficulty.
2. Development of the new project data mining methodology which is based on the top-down structuring of data.
3. Development of the new project estimation methodology that is based on the theory of projects.

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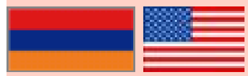

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