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Principles of Top-Down Quantitative Analysis of Projects

Part 1
State Equation of Projects and Project Change Analysis

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Abstract

Contemporary project management needs high-level quantitative representations of projects for their successful planning and execution. Such representation is possible on the basis of balance equations for the total effort of the project. On the basis of this balance equations can be obtained so-called state equation of projects.

This paper presents the derivation of this new equation that incorporates project size, project reuse, project duration, project total effort, team size, team productivity and project difficulty. In contrast to existing bottom-up methods of project presentation in the form of statistical relationships between project parameters this new top-down methodology is not a data driven and therefore is more accurate and stable.

Also the state equation of projects is used for change analysis and management.

Key words: Mathematical models of projects, state equation of projects, equilibrium of projects, project space, project change management, project goals and objectives.

Introduction

Traditionally project management is a field where qualitative methods dominate over the quantitative methods. Manager's intuition remains the main basis for decision making in project management. Domination of qualitative methods in project management over the quantitative methods has its deep reasons and explanations.

The main reason for this domination is that the contemporary project management rather an art than science.

Despite of the long history of quantitative methods in project management they cannot be considered as self-sufficient and independent field of knowledge by any means. In the best case they can be considered as a kind of supplement or addition to the traditional qualitative methods of project management.

The majority of contemporary quantitative methods in project management are the results and generalizations of project data mining. In fact each project data is a result of some passive experimentation which simply means that they cannot serve as a reliable source for extracting functional relationships between project parameters.

In order to do this in a correct way it is necessary to transform passive project data into the quasi-active project data. In its turn this transformation cannot be done without a top-down theoretical guidance in the form of data clustering.

Because of the absence of this kind of top-down theoretical guidance in the field of project management, usually passive project data is treated directly as if the data is a result of an active experimentation.

As a result of this kind of blind data processing project management area has generated a very unreliable set of empirical relationships for project estimation purposes.

Therefore these data dependent empirical relationships have low accuracy and as a whole they cannot serve as a basis for a self-sufficient quantitative project management.

Besides a considerable part of project parameters are direct functions of project's goals and objectives. In spite of this, methods of contemporary quantitative project management do not take into account for it.

As a result project estimation and prediction methods are in deep crisis.

The main reason for this is the absence of fundamental approaches to the problems of project quantitative description and analysis.

There is no a commonly established and unified mathematical view on the problems of project quantitative description.

Rather there are separate and uncoordinated attempts to describe different aspects of project planning and project execution by means of empirical simple models.

As a result project managers continue to use qualitative methods of project management only because the existing quantitative methods have unacceptable low accuracy.

In this situation the solution of the problem is the development of a top-down data independent mathematical theory of project management. The creation of high-level quantitative representations of projects is the first step in this process.

1. High-level quantitative representations of projects: State equation of projects

Different high-level presentations of project effort in the form of balance equations can serve as a basis for mathematical modeling of an arbitrary project. Such representation does not depend on the application field of the project because at that level of presentation all projects have the same set of parameters. These parameters are: Project total effort E, project duration T, project average staffing N_{av}, team productivity P, project size S, project reuse R and project difficulty D.

For the establishment of a balance equation between these parameters we can take advantage of two quantitative definitions of the project total effort [1].

First, let's define the quantity of work W related to the project as a product of total effort E and team productivity P

$$W = E * P \qquad (1)$$

The same quantity of work W can be defined as the product of project size S and project difficulty D

$$W = S * D \qquad (2)$$

From expressions (1) and (2) we can have a simple definition of the project total effort as

$$E = \frac{S * D}{P} \qquad (3)$$

The same total effort can be defined as the product of project duration T and project average staffing N_{av}

$$E = N_{av} * T \qquad (4)$$

Equalizing expressions (3) and (4) one can obtain the main balance or state equation of projects

$$N_{av} * T = \frac{S * D}{P} \qquad (5)$$

If there is some level of project reuse R then the new size of project will be $S - R$ and the corresponding state equation of project will have the form

$$N_{av} * T = \frac{(S - R) * D}{P} \qquad (6).$$

From here we can make transitions to the different forms of this equation.

In spite of their relatively simple form, these equations reflect an extremely complex behavior of interdependencies between parameters of the project.

Characteristic feature of these equations is that the change of the one of parameters leads to unpredictable chain changes of other parameters. This makes it impossible to utilize these equations for direct estimations of project parameters. For that purpose we need to develop special mathematical methods.

Let's consider some examples of these interdependencies and chain changes of project's parameters.

Example1. Change of project size S: For this analysis we can use expression (5). Assume instead of project size S in expression (5) we have a new size $S + \Delta S$, where ΔS is the change of project size. The question is that what consequences can have this change for the whole project. It is clear that changing project size we need to change project duration on some value ΔT and project staffing on another value ΔN_{av}. In its turn the change of project average staffing ΔN_{av} will result a new value of team productivity $P + \Delta P$ because of the changed interaction between the people. This will cause a serious uncertainty because the change ΔS will generate three new changes ΔT, ΔN_{av} and ΔP. This will result a transition of the project from one state to a new state in project space with a new balance equation

$$(N_{av} + \Delta N) * (T + \Delta T) = \frac{(S + \Delta S) * D}{P + \Delta P} \qquad (7)$$

To be able to manage the project work with ΔS change we need to know the values of changes ΔT, ΔN_{av} and ΔP. The solution of this problem is beyond of the capabilities of the contemporary quantitative project management because from a formal mathematical point of view we have one equation (7) and three ΔT, ΔN_{av} and ΔP unknowns. In fact in accordance with the equation (5) takes place a division of the increment of project size ΔS between three increments ΔT, ΔN_{av} and ΔP. To solve this problem we need two more equations or some assumptions about the extreme character of change trajectories in the project space.

The solution of this problem can be based on the statement that there exist fundamental relationships between project parameters and we are able to find out those relationships by adding some new principles to the basic state equation of projects.

Detailed discussion of this problem will be given in the subsequent sections of this paper.

Example2. Change of the duration of project T for constant size S and constant difficulty D of the project: In this case the change of project duration ΔT will lead to the change of the average staffing level of the project ΔN_{av}. In its turn the change ΔN_{av} will lead to the change of team productivity ΔP . Unlike the previous example in this case we have one equation (8) and two unknowns - ΔN_{av} and ΔP .

$$(N_{av} + \Delta N) * (T + \Delta T) = \frac{S * D}{P + \Delta P} \qquad (8)$$

There can be many other examples of project changes and all those belong to the field of project change analysis and change management. These project changes can be analyzed with the aid of state equations and they are applicable for the problems of project planning, project execution and project post mortem analysis and classification.

2. Project space

For quantitative representation of project space it is convenient to use the equation (6). It contains six high-level parameters and it is always possible to present projects in the three-dimensional space using three of them. For instance, we can present the state equation of projects in the form

$$N_{av} = \frac{(S - R) * D}{P * T} \qquad (9)$$

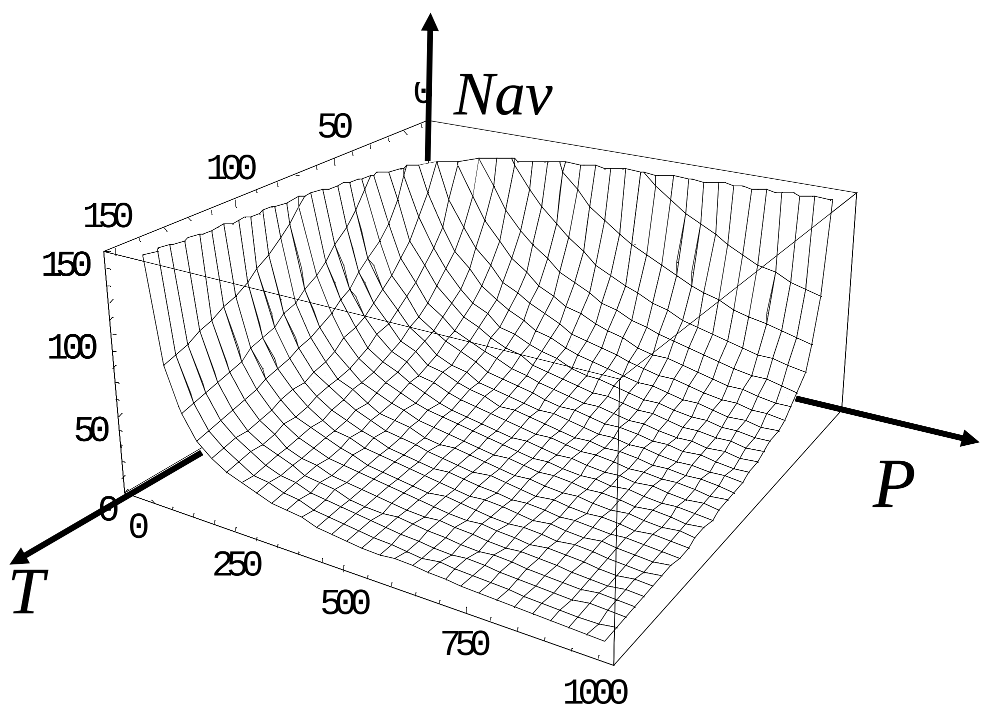

Fig.1 Project space that presents equation (9) for constant work complexity W

In accordance with this equation Fig.1 presents the project space for constant project complexity W

$$(S - R) * D = W = Const . \qquad (10)$$

In this picture the three axes are: staffing level N_{av}, project duration T and team productivity P.

As we stated above project complexity W is the amount of work (or work complexity) that needs to be done for completing the project. As we can see from expression (10) this parameter W consists of two parts.

The first part is $(S - R)$ and that is the size of the project. This part is completely human independent. The second part is the difficulty of work D. This parameter is human dependent and it is meaningful only in a binding with the productivity of human work [1]. As we can see from Fig.1 the work with constant size and difficulty can be presented as a three-dimensional hyperbolic surface. Each point on this surface has three coordinates and represents itself one potential realization of the project. Let's call it project point.

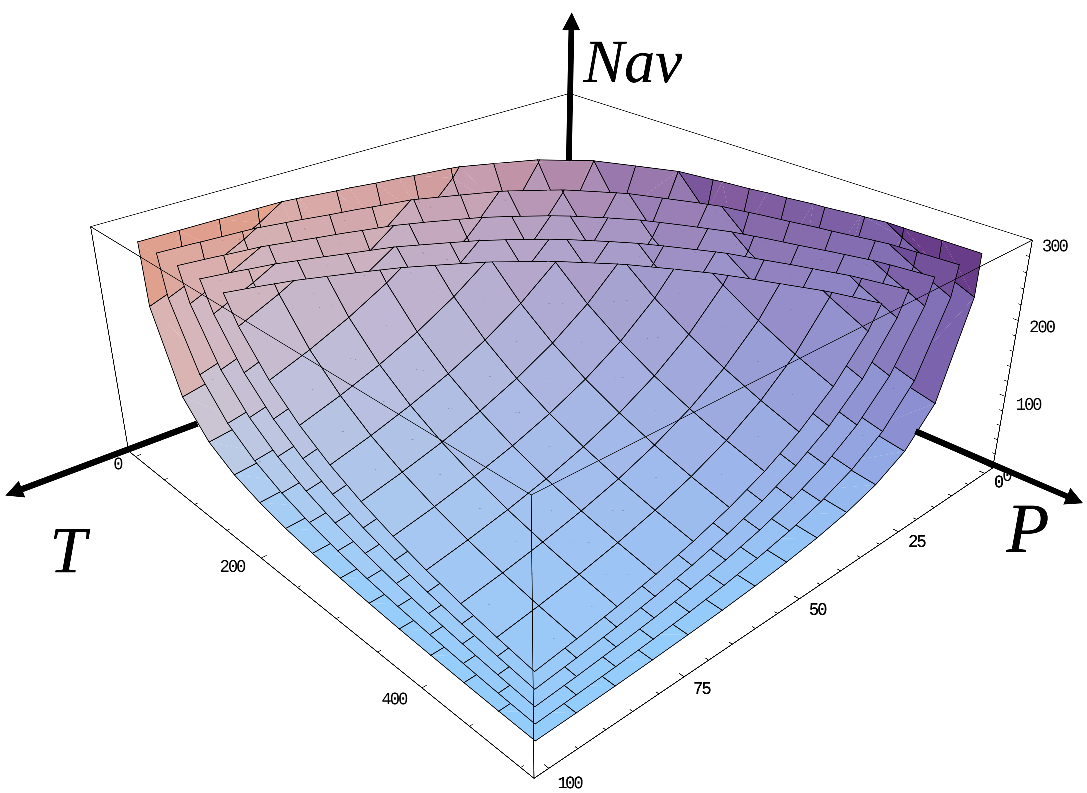

Fig.2 Graphical presentation of five different project surfaces for five options of the same projects with different sizes and work difficulties

Transition from one possible option of the project to another possible option occurs only along with some trajectories that lie on this surface. During these transitions project point moves from one position in the project space to a new position, continuing to be on the same project complexity surface with constant W .

For graphical presentation of project size and project difficulty changes it is possible to use multiple surfaces in the same project space. In Fig.2 we can see five project surfaces that represent five different sizes and work complexities of a project.

With each change of project size or its difficulty the project point shifts from one project surface to another. That shift corresponds to the new amount of the work complexity because any change of size or work difficulty changes the needed amount of work to complete the project. It is remarkable that this transition occurs not in a random fashion but along with some extreme trajectories in the project space. The study and finding of these extreme trajectories is the central problem of the quantitative project management.

Those extreme trajectories are the results of combination of the state equation of projects with project related extremum principles. These principles are closely related to the effort, cost and risk minimization.

3. Analysis of the state equation of projects

The state equation of projects (6) establishes an obvious balance between project parameters and it is true for all cases. But alone this equation is unable to describe functional relationships between these parameters of a project unambiguously. Referring to the graphical language, the equation of state describes all possible trajectories in the project space. But for an unambiguous quantitative presentation of functional relationships between the parameters of a project we need only one of these infinite number of trajectories that satisfy some additional conditions. These may be the conditions of constancy of certain parameters of the project or it may be a requirement of an extremum for the one of its system-level performance indicators. For example, it can be the requirement of the minimum of the total project cost.

But we always have to remember that the artificial requirement of the constancy of certain parameters of the project, in order to obtain functional relationships between other parameters, is fraught with potential errors.

A classical example of such an error may be the wrong interpretation of the functional relationship between team productivity P and team size N_{av}.

As can be seen from the equation of state (6), with the constancy of the other terms in it the functional relationship between these parameters takes the following form.

$$N_{av} * P = \frac{(S - R) * D}{T} = Const \qquad (11)$$

This means that the relationship between team productivity and team size is hyperbolic in nature, which is contrary to the simple logic. This is equivalent to the statement that working in teams consisting of 100 members, people will have several times less productivity in comparison with the productivity when working in a team consisting of 30 persons.

The main reason for this false statement is the artificial maintenance of constancy of the project duration T . In reality, a constant can only be the size of the project, and all the rest, when you modify one of them, changed freely, but in harmony with special fundamental laws. The essence of new methods of quantitative project management is the identification of these fundamental laws.

4. Change of project duration with constant project size

Let's continue analysis with Example2 for establishing relationship between the changes of project duration, the number of people and the productivity of their work at a constant size and difficulty of the project.

From expression (5) for project duration as a function of the two variables P and N_{av} we can have

$$T = \frac{S * D}{P * N_{av}} \qquad (12)$$

From here an arbitrary small change ΔT of the project duration can be presented as

$$\Delta T = \frac{\partial T}{\partial N_{av}} \Delta N_{av} + \frac{\partial T}{\partial P} \Delta P \qquad (13)$$

Also for small changes it can be presented as the differential of the project duration T

$$dT = \frac{\partial T}{\partial N_{av}} dN_{av} + \frac{\partial T}{\partial P} dP \qquad (14)$$

Partial derivatives $\frac{\partial T}{\partial N_{av}}$ and $\frac{\partial T}{\partial P}$ can be found from the expression (12).

$$\frac{\partial T}{\partial N_{av}} = -\frac{S * D}{P N_{av}^2} \qquad (15)$$

and
$$\frac{\partial T}{\partial P} = -\frac{S*D}{N_{av}P^2} \qquad (16)$$

By substituting (15) and (16) into the expression for the differential (14) we can have

$$dT = -\frac{S*D}{PN_{av}^2}dN_{av} - \frac{S*D}{N_{av}P^2}dP \qquad (17)$$

Taking into account that $T = \frac{S*D}{P*N_{av}}$, from the expression (17) we can obtain the main relationship between the changes of T, P and N_{av}.

$$\frac{dT}{T} = -\frac{dN_{av}}{N_{av}} - \frac{dP}{P} \qquad (18)$$

This expression indicates that the relative change of the project duration is being split between the relative changes of team size and team productivity. Attention should be paid to the signs of increments in this expression. If the project is the same and duration T increases then N_{av} decreases and as a consequence P increases too and vice versa.

Also we can present the expression (18) in the form $\frac{dT}{T} + \frac{dN_{av}}{N_{av}} + \frac{dP}{P} = 0$, which means that the sum of all the relative changes of the project is 0.

5. Project goals and the change of project duration

Let's analyze the project duration change from the point of view of project goals and priorities in the case of the constant project size.

For this analysis let's introduce a new variable – the power of the team H as the product of team size N_{av} and team productivity P

$$H = N_{av} * P \qquad (19)$$

This introduction of the new variable of the team power H allows represent the project complexity W in the form of the product of the team power and project duration

$$T * H = W \qquad (20)$$

This expression has its direct analogue in mechanics, where the work is the product of the power and time. Continuing the analogy with mechanics, easily notice that the number of people N_{av} is similar to the mechanical force and the mechanical speed is the analogue of the team productivity P.

Let's continue the change analysis with constant project size and complexity of work W. Corresponding two-dimensional project space is presented in Fig.3. In this picture we have a hyperbola that corresponds to the expression (20).

Assume the initial plan of some project was to accomplish a project with work complexity W within T_1 months and with a team power H_1.

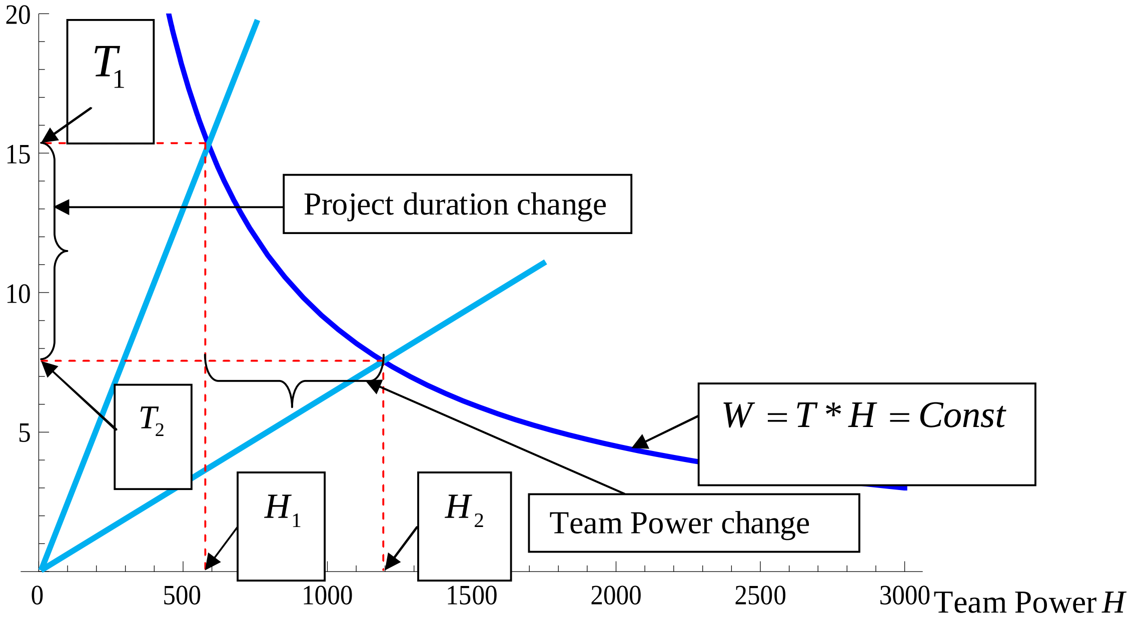

Fig.3 Two-dimensional project space for change analysis

Suppose on the basis of some considerations, we decided to shorten the duration of the project from T_1 to T_2. This means that the project point will be displaced from (T_1, H_1) to the new point (T_2, H_2). Along with other things these two points are reflecting project high-level goals and priorities. In most cases these priorities are related to the project cost and time considerations. For example, shortening the project duration makes it more expensive.

Those priorities quantitatively can be characterized by the ratios $r_1 = \frac{T_1}{H_1}$ and $r_2 = \frac{T_2}{H_2}$.

Two straight lines in this picture are the lines of equal priorities of project goals.

The equation of the first priority line is $T = \frac{T_1}{H_1} H$ and the equation for the second priority line is $T = \frac{T_2}{H_2} H$.

Thus in fact changing some parameters of projects we simply reflect the changes in their high-level priorities.

So in Fig.3 we have two changes $\Delta T = T_1 - T_2$ and $\Delta H = H_1 - H_2$. If these changes are small enough we can present the relationship between them in the following form

$$dT = -\frac{W}{H^2} dH . \quad (21)$$

6. **Change of project duration with variable project size and complexity**

Let's consider two specific cases with the variable project complexity.

1. Project complexity change without changing the priorities of its high-level goals.
2. Project complexity change and simultaneous change of priorities of high-level project goals.

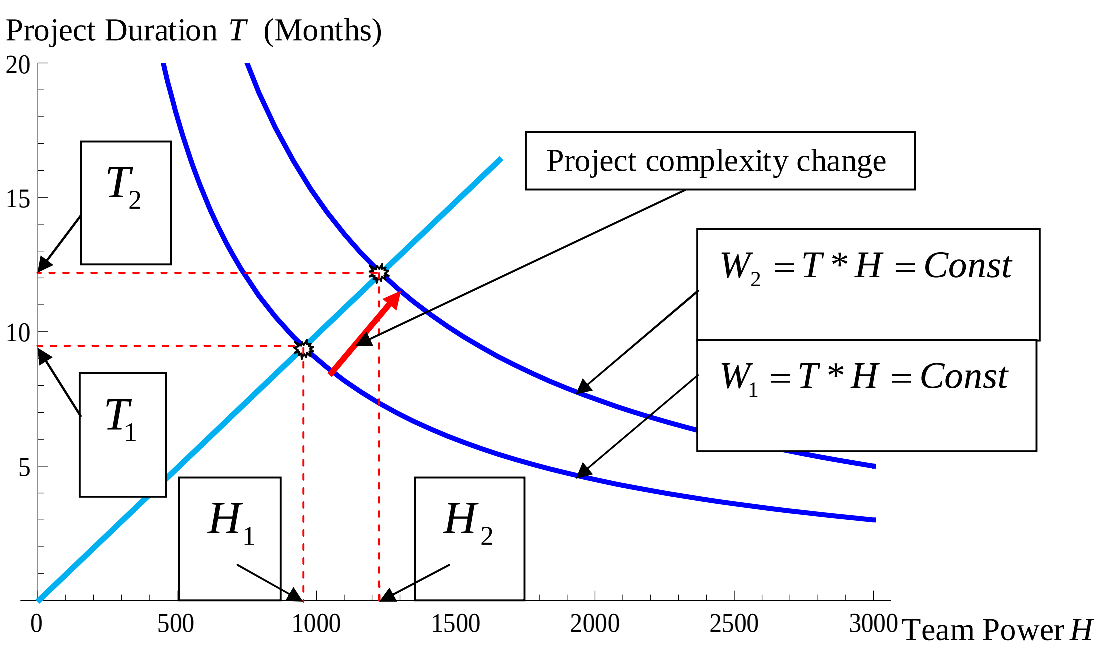

Fig.4 Project complexity change without changing the priorities of its high-level goals

Let's analyze these cases separately.

1. In the case of the change of the project's complexity only we will have a two-dimensional project space presented in Fig.4.

Since the priorities of project objectives are not changed then $r_1 = \frac{T_1}{H_1} = r_2 = \frac{T_2}{H_2} = r$

In its turn the project change can be presented as

$$dW = HdT + TdH \qquad (22)$$

From expression (22) we can find changes of project duration T and team power H

$$dT = T_2 - T_1 = \frac{dW}{H} - rdH \qquad (23)$$

$$dH = H_2 - H_1 = \frac{dW}{T} - \frac{dT}{r} \qquad (24)$$

2. Simultaneous project complexity and priority change is illustrated in the Fig.5.

The original project point is (H_1, T_1). Because of the project complexity change $\Delta W = W_2 - W_1$, the project point moves from (H_1, T_1) to (H_2, T_2).

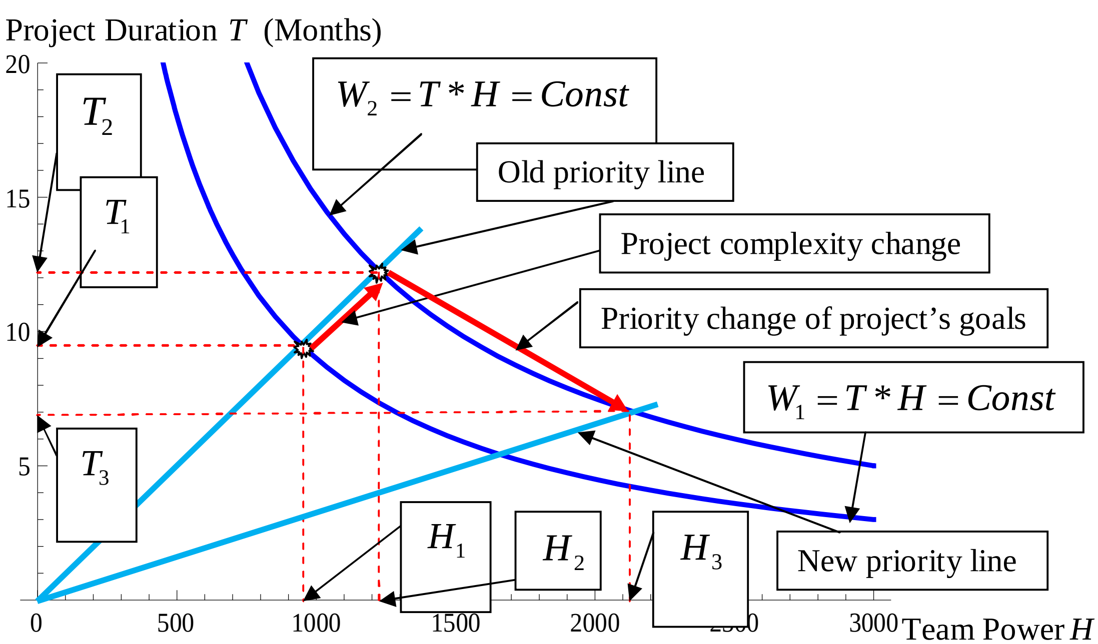

Fig.5 Project complexity change with changing the priorities of its high-level goals

Then because of the priority change of the project goals project duration shortens becoming T_3. In order to overcome W_2 complexity within T_3 months team power needs to be H_3 . So with the second change the project point moves from (H_2, T_2) to (H_3, T_3).Thus, due to two changes (project complexity change and change of priorities of project goals) the duration of the project will be reduced from T_1 to T_3, and the team power will be raised from H_1 to H_3.

7. Team Power as a source for project change analysis

Team power can be presented in the following two forms:

$$H = N_{av} * P \quad \text{and} \qquad H = \frac{W}{T} = \frac{S * D}{T} \qquad (25)$$

From here for the change of team power we can have two expressions:

$$dH = P * dN_{av} + N_{av} * dP \qquad (26)$$

$$dH = \frac{T * dW - W * dT}{T^2} \qquad (27)$$

Taking into account that $W = S * D$ and $dW = D * dS + S * dD$, from (27) we can have

$$dH = \frac{T * D * dS + T * S * dD - S * D * dT}{T^2} \qquad (28)$$

Equalizing two (26) and (28) expressions for dH one can obtain

$$T^2 P * dN_{av} + T^2 N_{av} * dP = T * D * dS + T * S * dD - S * D * dT \qquad (29)$$

This new expression clearly shows that there is a very complicated relationship between changes of a project. Also any change of the one of project parameters can cause changes of other parameters. Mathematically expression (29) is one equation with five unknown changes. Therefore for its solution we need additional conditions.

Taking into account that $E = N_{av} * T$ we can have another form of expression (29) that contains project total effort too

$$T^2 P * dN_{av} + T * E * dP = T * D * dS + T * S * dD - S * D * dT . \qquad (30)$$

8. Three-dimensional graphical presentation of project changes

Project change management needs adequate presentations of the changes of project parameters in the project space. If some project has a constant size and constant difficulty, any change in its parameters (N_{av}, T, P) leads to the displacement of the project point on the same surface. These changes are represented in Fig.6 by red curves. In this picture we have two surfaces that represent two constant values of the project complexity. Also here we have five project points that represent five different states of the same project. Green curves are presenting project transitions from one complexity value to another value of it.

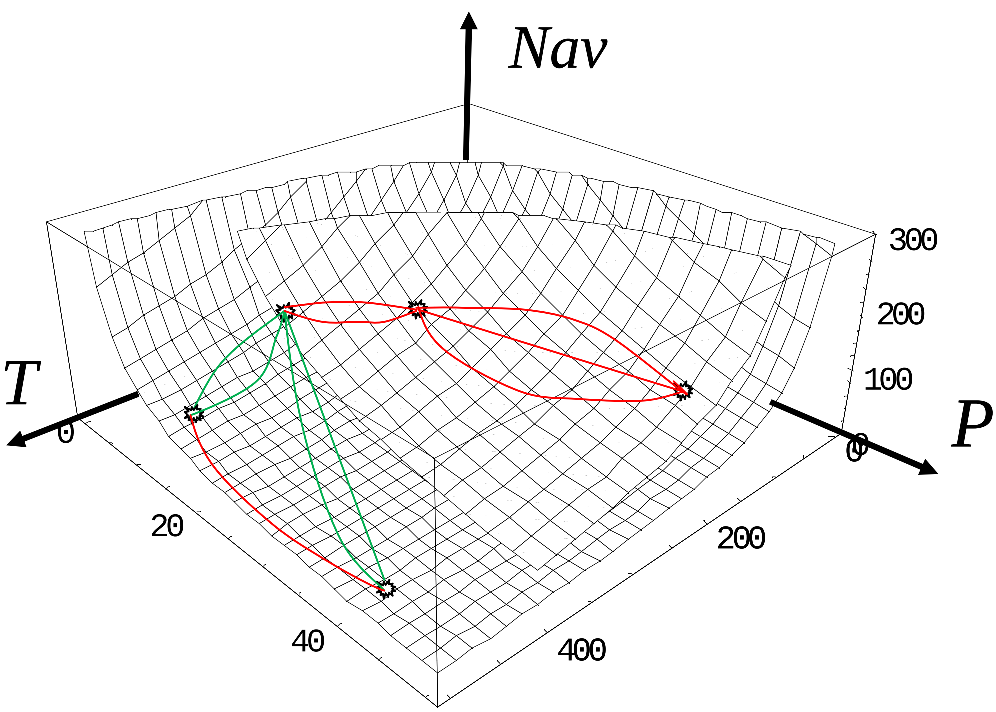

Fig.6 Five project points in the project space with transition trajectories between them

So any change of project parameters leads to the motion of the project point from one equilibrium state of the project to another equilibrium state of it. There is infinite number of transition trajectories between any two project points in the project space. Among these there are such trajectories, which correspond to extreme values of system performance indicators.

These performance indicators are closely related to the requirements of minimum development cost (minC), minimum total effort (minE), minimum risk (minR), minimum project cycle time (minT), etc. In their turn all system-level performance indicators of projects are the functions of transition trajectories between project points in the project space. Moreover, the extreme values of system performance indicators correspond to extremum trajectories in the project space. These extremum trajectories contain all the functional dependencies between project parameters.

Thus, on the one hand, all trajectories are contained in the equation of state projects (6), on the other hand the system performance indicators of the projects are functions of the same trajectories.
This means that the extremum trajectories in project space and functional relationships between project parameters can be found through the joint solution of the state equation of projects with extremum conditions or system-level performance indicators of the project.

Depending on the system-level priorities of project goals the trade-off between them can vary in large limits. Each specific trade-off between the objectives of the project will produce its own extremum trajectories in the project space. Correspondingly each new trade-off between system-level performance indicators will produce its own functional relationships between project parameters.

Project goal based trajectories are not the only extremum trajectories in the project space.
In this sense the gradients of project parameters in the project space potentially can play an important role too.

9. Conclusions and future work

1. The state equation of projects is a generalized high-level presentation of an arbitrary project.
2. The state equation of projects is an application invariant presentation of projects.
3. The state equation of projects in conjunction with project goals and objectives can serve as a basis for the analytical derivation of the functional relationships between project parameters.
4. This indicates that there is a possibility to derive all the functional relationships between the project parameters without any bottom-up project data processing.
5. The state equation of projects is the generalization of project related common knowledge therefore in some sense it is able to replace the project data.
6. State equation based project change analysis can open new ways for effective change management in project planning, project execution and project post mortem analysis and classification.
7. Project goals and their system-level priorities have a direct impact on the project parameters.
8. Quantitative analysis based on the project state equation in conjunction with quantitative presentations of project goals and their priorities can open new ways for the analysis of large projects and megaprojects.

Future work is connected with:

1. Non-linear scaling of projects as functions of their size and difficulty.
2. Comprehensive quantitative analysis of "One size does not fit all" problem [2, 3].
3. Project goals and their priorities as functions of project size.
4. Change of project reuse.
5. Earned Value Management and state equation of projects.
6. Team productivity as a function of project size and its difficulty.
7. Crisis in contemporary project data mining and project estimation.

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