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Reliability of Projects: A Quantitative Approach
Part 1
Reliability and Productivity Models of the Elements of Work Flows

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Abstract

Classical theory of reliability is applicable for the project's reliability analysis. The main idea behind the mathematical modeling of the project's reliability is to represent the entire project as a mixed system of parallel-serial human actions. Every human action, depending on the difficulty of the problems under investigation, with some probability can be successful or unsuccessful. Combining these probabilities with the structure of parallel-serial human actions, it is always possible to assess the probability of success or failure for the entire project.

In order to solve these problems we need to deal with the extended reliability block diagrams of projects that include communication and control related human actions.

This work consists of two parts.

The first part discusses issues related to the modeling of reliability and productivity of the elements of the reliability block diagrams of projects in the form of serial and parallel human actions. The second part of this work will be devoted to system-level issues of the assessment of project's reliability parameters.

Introduction

Clearly the problem of the projects reliability is a critical unresolved issue in the modern project management. According to many sources the financial losses from unfinished projects comprises hundreds of billions of dollars. In particular, this problem is very acute in software engineering, where the direct failure or delays of projects with significant financial overruns are a real disaster for the industry.

One reason for this state of affairs in project management is that the quantitative theory of project reliability is not sufficiently advanced and the purpose of this work is to fill this gap in some extends.

The basis of this work is the assertion that the classical theory of reliability of systems is fully applicable for analyzing the reliability of the projects and their failure rate.

The main idea behind the mathematical modeling of the reliability of the projects is to represent the entire project as a mixed system of parallel-serial human actions. Every human action, depending on the difficulty of the problems under investigation, with some probability can be successful or unsuccessful. Combining these probabilities with the structure of parallel-serial human actions, it is always possible to assess the probability of success or failure for the entire project.

Another important claim, for analyzing the reliability of the projects is that each schedule of a project can have different reliability block diagrams in the form of mixed systems of parallel-serial human actions with different levels of detailed presentations.

Discussion of the ways to address the issue

Since the elements of the reliability block diagrams of projects are the individual and group human actions, the research in this area can be divided into two phases.

1. Mathematical modeling of the elements of the reliability block diagrams of projects in the form of serial and parallel human actions.

2. Quantitative analysis of the system-level reliability block diagrams of projects as a structural presentation of the individual and group human actions.

Accordingly, in the first part of this work, we study issues related to the modeling of reliability and productivity of the elements of work flows, while the second part – is the modeling of the reliability and productivity of projects as a whole.

Decision dynamics in the course of projects

The process of designing is a flow of human decisions, and over time, qualitative and quantitative composition of the made decisions varies greatly. In the initial stages of the project are made responsible decisions that are crucial to the viability of projects and related diversified risks.

The decision process can be characterized by its density $m(t)$, which is the number of the project related decisions per unit time. In quantitative terms, this function grows up to a certain peak and then falls as we approach to the completion of the project (Fig.1).

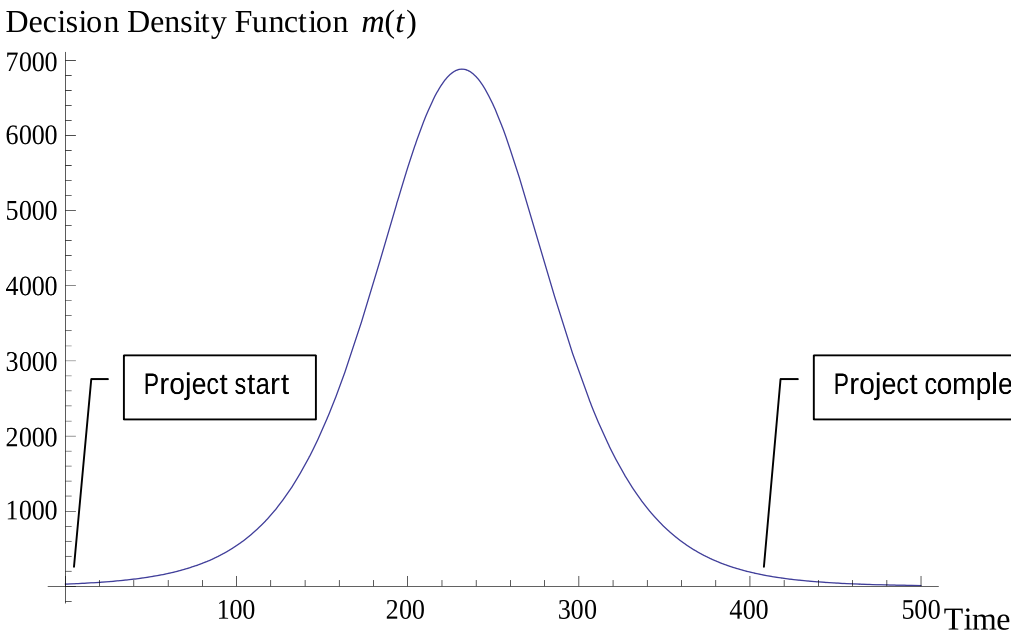

Fig.1 Decision density function in the course of projects

The integral number of the made decisions in the course of projects $M(t)$ is presented in Fig.2.

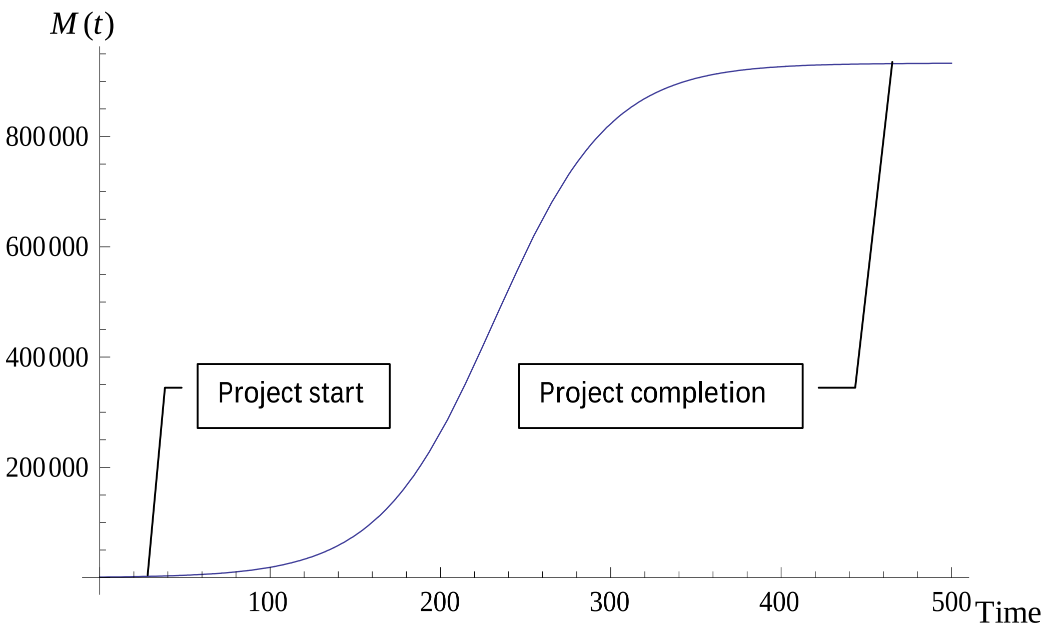

Fig.2 Integral number of made decisions in the course of projects

Thus there is an obvious relationship between two functions $M(t)$ and $m(t)$

$$\frac{dM(t)}{dt} = m(t) \qquad (1)$$

Besides, depending on the strategy of the organization of work, the peak for the decision density function can be achieved earlier or later in the course of projects. Accordingly, there are projects with early peak and late peak by the number of made decisions (Fig.3).

In order to describe this process quantitatively it is possible to use different growth curves for the function $M(t)$ [2], as in all cases we are dealing with increasing of the total number of decisions as a function of time (Fig.2).

The most suitable for this purpose is the Richard's differential equation and the Richard's function [3]. This equation has a number of parameters, suitable for flexible presentation of the different forms of the project effort distribution over time.

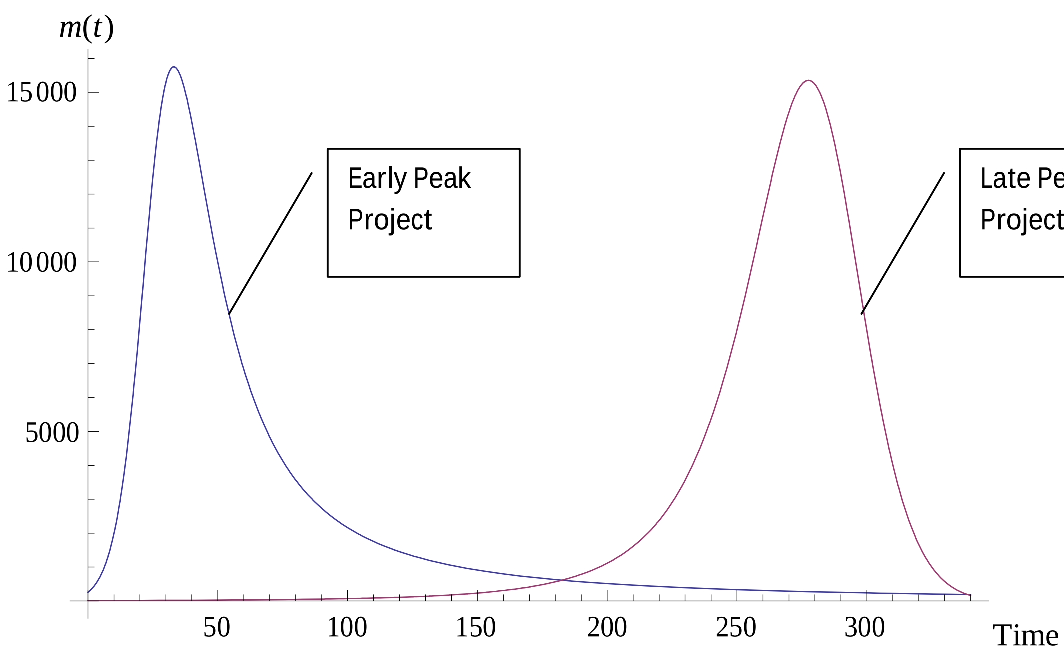

Fig.3 Decision density functions for early and late projects

Reliability and Risk related problems of projects with early and late peaks

Consider the problems associated with the early and late peaks of projects in terms of their reliability.

For discussion of this problem some circumstances have an important role, for analysis of which let's consider the distribution of the project effort over time.

1. Project plan with an early peak facilitates early parallelism of project works, which, in certain circumstances could have unintended consequences.

For example undertaking of works without the completion of some previous works can lead to unnecessary repetition and to increase of the project effort which leads to the increase of the number of total errors. In addition, such an increase in the number of errors due to the increased parallelism of work leads to the non-linear increase in the number of undiscovered errors too, that later may have undesirable consequences.

The reason here is that each next step in the project work is a natural verification for the previous, already completed works. If the next work begins without completing the previous work, it violates the natural course of verification, leading to additional undetected errors, which have an undesirable feature of branching too.

2. Projects with the late peak of the staffing profile facilitate the late parallelism of works, resulting in a rush, in turn contributing to the emergence of additional errors. In addition, with the late peak the main volume of work is concentrated at the end of the project. This means that the same thing happens with the main mass of errors, so, as the number of errors is in proportion to the volume of the work done. It is important that in order to correct errors it is necessary to spend certain time. This means that if there is a late rush of errors, the likelihood that the tails of the errors blow out for the designated time limits will increase. This will lead to the additional project delays.

Besides, the late peak means late project integration with all its undesirable consequences.

At any stage of the project works the number of made errors can be divided into two groups, the first of which is the group of the small number of high-level errors and the second is the large number of minor errors pertaining to the detailed design.

In this sense, there are fundamental differences between the small number of high level errors, associated with the structural design and the many relatively insignificant errors, associated with the detailed design.

The above considerations may serve as a basis for the development of the methods of analysis and synthesis of the effort distribution over time, which is the staffing profile of projects.

This analysis shows that, if you leave aside the reuse related issues based solely on the project reliability related considerations it is possible to synthesize the project staffing profile. Quantitative analysis of this problem may have a major impact on the progress in the theory of project management in general.

Equally important are the issues related to the synthesis of high-level staffing profiles of projects. It is therefore important to focus attention of the researchers on the following issues:

- Application of Richard's differential equation for the quantitative presentation of various project staffing profiles.
- Establishment of functional relationships between project goals and the forms of project staffing profiles.
- High-level synthesis of the project staffing profiles based on the goals and objectives of projects, including multi-objective optimization related issues.
- Study of the Rayleigh curve from the point of view of high-level analysis and synthesis of the project's staffing profile, focusing an attention on the inability of this curve to adequately reflect the effort distribution at the starting points of projects.

Relationships between decision-making process and project parameters

The number of decisions over the project time is closely related with the other parameters of projects. Because of the measurement uncertainties of the number of decisions over time, for the project estimation purposes we need to establish interrelationships between the characteristics of the decision making process and other measurable project parameters. First of all it is necessary to establish those relationships between decision density function and two effort characteristics: total effort of a project E as a function of project time and the effort distribution over time, which is the project staffing profile $n(t)$. There is an obvious relationship between these two parameters [5].

$$\frac{dE}{dt} = n(t) \qquad (2)$$

Project total effort E and project staffing profile $n(t)$ are related to the other important project parameter – complexity W, which is the product of the project size S and its difficulty D [6]

$$W = S * D \qquad (3)$$

Project total effort E and its derivative $n(t)$ have functional relationships with the project complexity density over time w

$$w = \frac{dW}{dt} \qquad (4)$$

In their turn project effort and its distribution over time can serve as a basis for the obtaining of the total number of human actions and their distribution over time [Fig.4].
If the reliability of a project is a functional that depends on the system-level parameters (total effort, project complexity, project size, etc.) and their distributions over time, then its analysis

reduces to the classical variational problem when the basic functional depends on some function and its derivative [7].

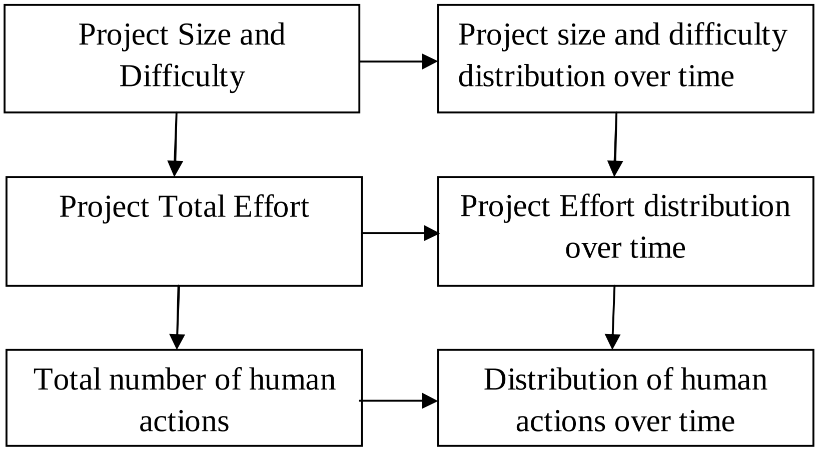

Fig. 4 Interrelationships between project parameters for project reliability analysis

In any case cumulative project complexity, project cumulative effort and the cumulative number of project decisions are the growing functions of time. Therefore the Richard's differential equation can be applied for their quantitative description.

In particular for the presentation of the current value E of the project's total effort as a growing function of time the Richard's differential equation has the form

$$\frac{dE}{dt} = \frac{a}{\sigma} E\left(1 - \frac{E}{E_{\max}}\right)^{\sigma} . \qquad (5)$$

Here $\frac{a}{\sigma}$ is the effort growth rate and σ reflects the asymmetric behavior of the growth process.

$E_{\max}$ - is the total effort of the whole project.

The important feature of this equation is that it allows reflecting the non-symmetrical growth patterns for the project effort and for the project staffing profile, thus reflecting the behavior of the early and late peak projects.

Besides it allows making transitions to the other differential equations that reflect the behavior of the cumulative project complexity W and cumulative number of project decisions M .

Structure of project decisions and the risk strategy

For further analysis of the circumstances associated with the project risks the decision density function should be further structured, since it is known that the various project decisions differ in the extent of their potential impact on the parameters of risk and other system-level parameters of projects.

So this process has an additional dimension too because at any moment of time decisions differ by their potential impact on the project risks.

In order to account for this additional dimension of the decision making process let's introduce a new function $h(r,t)$ which has the following relationship with the decision density function

$$m(t) = \int_0^\infty h(r,t)dr \quad (6)$$

This new function $h(r,t)$ represents itself the distribution of the number of decisions at the time t by the potential impact of the maid decisions on the project risk (Fig.5).

The nature of this distribution depends on the nature of work performed by human beings. In carrying out simple works this distribution is concentrated in the area of the relatively small values of risk.

Increasing complexity of the human work is accompanied by the emergence of a more complex structure of human actions, due to differences in the degree of their influence on the system characteristics of the project.

For example, if for the simple human actions the preliminary planning of work and the accompanying discussions during its execution are not so important, for the difficult and complex work they are simply a necessity.

For complex human activities the feasibility analysis of work becoming a task of higher priority, which leads to the emergence of more complex flows of work.

The direct consequence of this kind of structuring of human work is the division of design works into the high-level design and detailed design phases.

The peculiarity of the decision-making for complex works is that among the huge number of decisions only a limited number of them have an important impact on the system-level characteristics of projects, including their characteristics associated with risk.

Usually $h(r,t)$ is a quickly falling function of risk. Fig.5 illustrates this function for an arbitrary project time t_1.

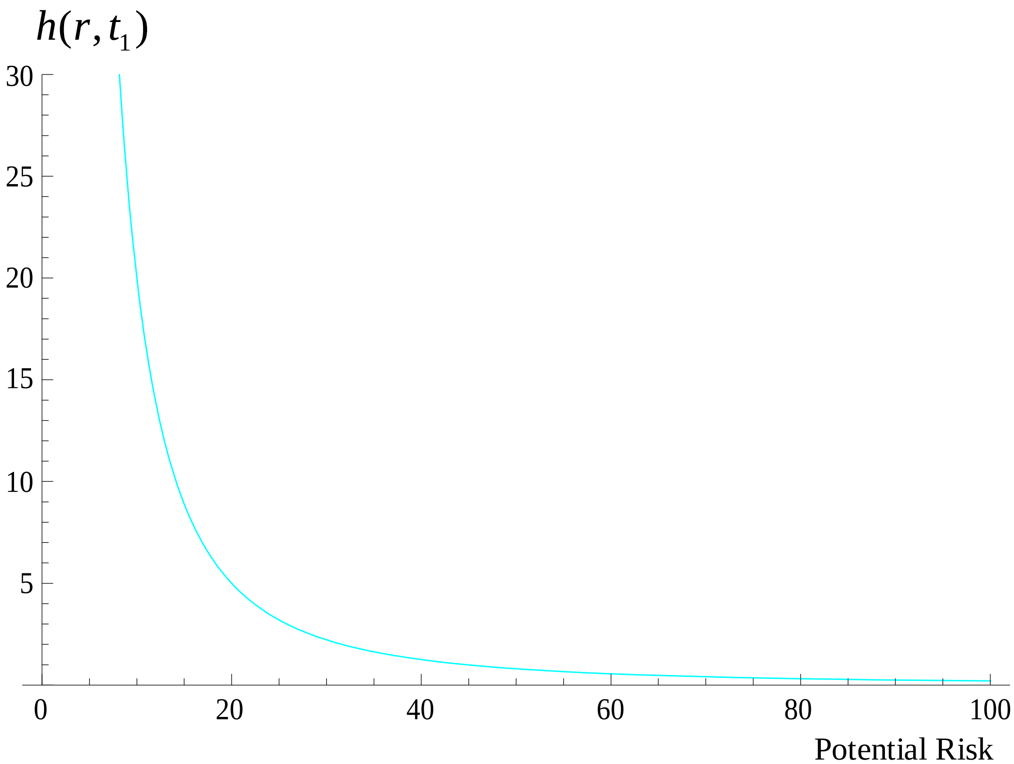

Fig.5 Distribution of the number of project decisions by their potential impact on the project risks for an arbitrary project time t_1

Risk oriented design strategy in the first place usually deals with those few decisions that are on the list of feasible problems and at the same time have the greatest potential impact on the project risk.

This behavior of the function $h(r,t)$ is a reflection of the fact that the number of tasks with the moderate complexity in project works dominates over the number of very complex tasks.

This problem has direct links to the power laws and strict analogies to known problems of the enterprises size distribution, people wealth distribution, and many others.

Reliability and productivity block diagrams of projects

Gantt charts and other detailed presentations of projects can serve as reliability and productivity block diagrams of projects. But from the point of view of reliability and productivity analyses there are missing portions of work in these presentations of project work.

There are many additional human actions related to the communication and control activities in the more detailed work flows and their reliability must be accounted for, for more accurate project planning and execution. Therefore for reliability and productivity analyses of projects it is necessary to make a transition from the ordinary work block diagrams to the block diagrams of project related human actions including technical reviews, information hiding, misleading information, superficial testing of work pieces, etc. For the accomplishment of this task approach presented in [4] can serve as a good guide.

Productivity of the serial connection of human actions

Assume we have a simple workflow presented in Fig.6. It is a serial work consisting of two serial portions with productivities P_1 and P_2. Also we have the total effort E as a sum of the efforts for these two portions

$$E = E_1 + E_2 \qquad (7)$$

The goal is to find out the resulting productivity P for the workflow with two serial portions of work.

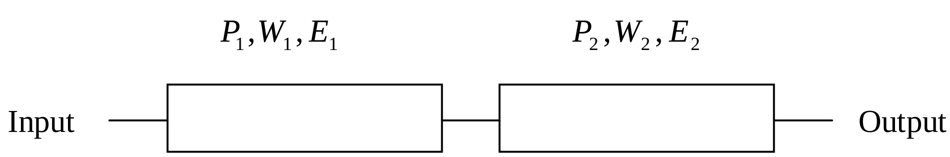

Fig.6 Simple work flow consisting of two serial stages

The total amount of work can be calculated as

$$W = P * E. \qquad (8)$$

Assuming that the work amount to complete the first part of work is $W_1 = k * W$ and correspondingly

$$W_2 = (1-k)W, \qquad (9)$$

we can have $W_1 = kW = E_1 P_1$ and $W_2 = (1-k)W = E_2 P_2$.
Defining E_1 and E_2 and substituting their values

$$E_1 = \frac{W_1}{P_1} = \frac{kW}{P_1} \quad (10)$$

and

$$E_2 = \frac{W_2}{P_2} = \frac{(1-k)W}{P_2} \quad (11)$$

into the equation (7) we can have

$$\frac{W}{P} = \frac{kW}{P_1} + \frac{(1-k)W}{P_2} \quad . \quad (12)$$

From here we can obtain

$$\frac{1}{P} = \frac{k}{P_1} + \frac{(1-k)}{P_2} = \frac{kP_2 + (1-k)P_1}{P_1 P_2} . \quad (13)$$

As a result we can have the following rule for two serial productivities

$$P = \frac{k}{P_1} + \frac{(1-k)}{P_2} = \frac{P_1 P_2}{(1-k)P_1 + kP_2} . \quad (14)$$

In the particular case when $P_1 = P_2$ we will have

$$P = \frac{P_1 P_1}{(1-k)P_1 + kP_1} = P_1 = P_2 . \quad (15)$$

This means that the resulting productivity for the whole workflow is equal to the productivities of the serial portions of work.

We can generalize the results for serial works with arbitrary number n of its serial portions. From (13) we can have

$$\frac{1}{P} = \frac{k_1}{P_1} + \frac{k_2}{P_2} + \ldots \frac{k_i}{P_i} + \ldots + \frac{k_n}{P_n} = \sum_{i=1}^{n} \frac{k_i}{P_i} \quad (16)$$

where

$$\sum_{i=1}^{n} k_i = 1$$

Let's apply the above results for two phase human work analysis using methods of "human effort dynamics" from [1].

Assume the developer who is performing some work with complexity W, has timing and productivity characteristics, presented in Fig.7 and Fig.8.

Duration of Human Action

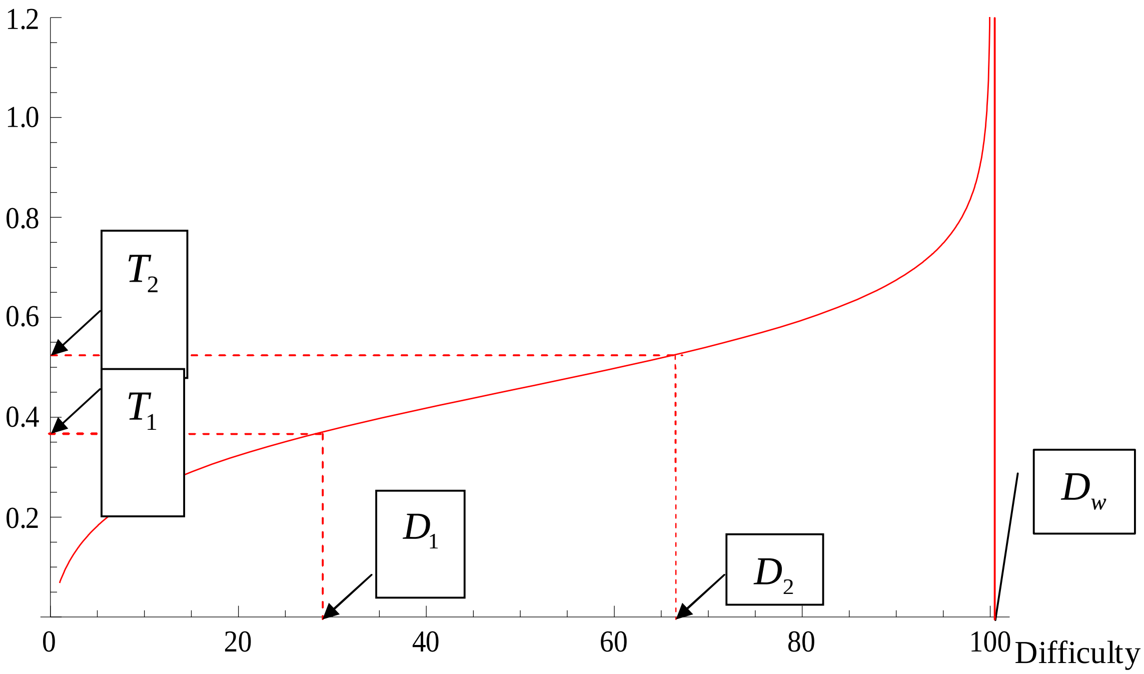

Fig.7 Timing of human actions as a function of the action's difficulty

Work W consists of two phases with the complexities W_1 and W_2 and $W = W_1 + W_2$.
In its turn the work W_1 has S_1 size and D_1 difficulty: $W_1 = S_1 D_1$. Similarly $W_2 = S_2 D_2$.

For this example $k_1 = \frac{W_1}{W}$ and $k_2 = \frac{W_2}{W}$ therefore

$$\frac{1}{P}=\frac{k_1}{P_1}+\frac{k_2}{P_2}. \qquad (17)$$

For the first phase of work the state equation for work duration has the form [1]

$$T_1=\frac{S_1D_1}{P_1} \qquad (18)$$

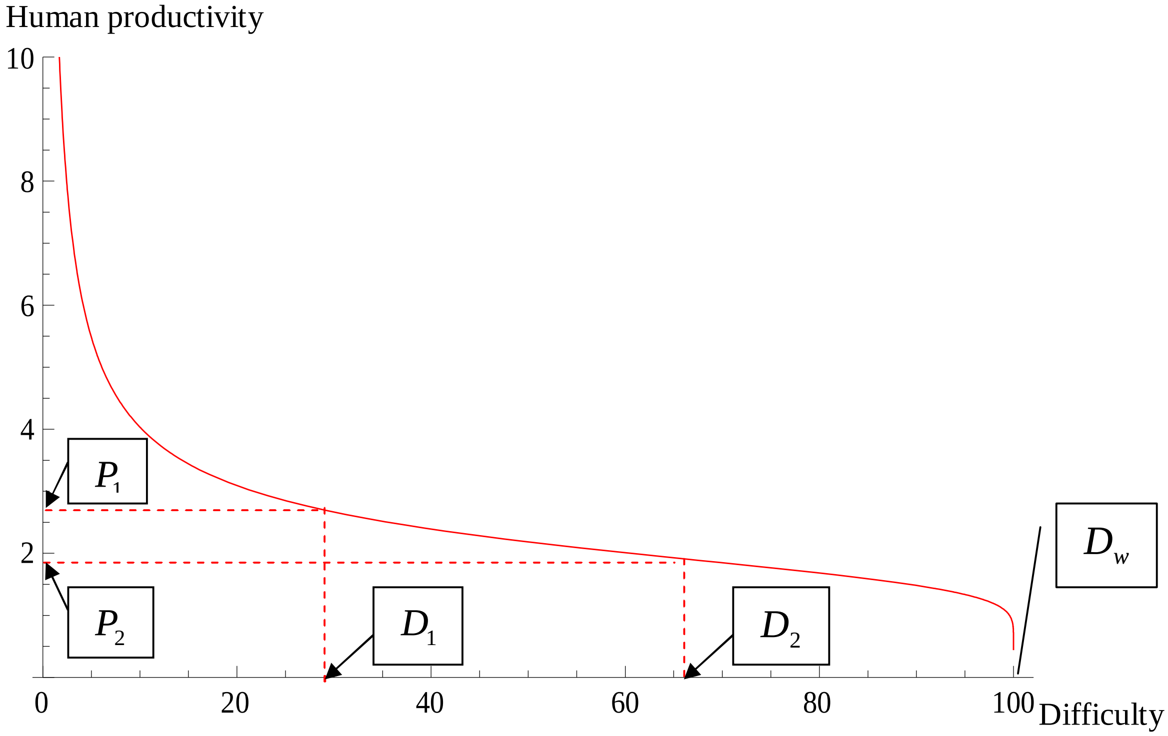

Fig.8 Human productivity as a function of the action's difficulty

From here we can have

$$P_1=\frac{S_1D_1}{T_1}. \qquad (19)$$

Similarly for the second phase of work we will have

$$T_2=\frac{S_2D_2}{P_2} \qquad (20)$$

and $$P_2 = \frac{S_2 D_2}{T_2} \qquad (21)$$

Substituting P_1 and P_2 from (19) and (21) correspondingly into the expression (17) we can have

$$\frac{1}{P} = \frac{k_1 T_1 S_2 D_2 + k_2 T_2 S_1 D_1}{S_1 S_2 D_1 D_2}. \qquad (22)$$

From here we can obtain

$$P = \frac{S_1 S_2 D_1 D_2}{k_1 T_1 S_2 D_2 + k_2 T_2 S_1 D_1} \qquad (23)$$

Combining the above results with the probabilistic assessments of the work success from [8] it is possible to use them for project's reliability analysis.

Probabilistic analysis of the duration of parallel work

Assessment of the duration of the people parallel work is a mandatory part of the general assessment of the duration of projects. The work of each individual has a probabilistic nature, which affects the parallel work of people.

For the sake of simplicity let's consider the work of two parallel working developers. Assume according to their schedule each of them has to complete the work at time t correspondingly with density functions $f_1(t)$ and $f_2(t)$. This means that independently the first developer can complete his work at time t with probability

$$\Pr ob_1 = F_1(t) = \int_0^t f_1(\vartheta) d\vartheta. \qquad (24)$$

For the second developer to complete the work at the time t independently from the first developer is

$$\Pr ob_2 = F_2(t) = \int_0^t f_2(\vartheta) d\vartheta. \qquad (25)$$

The situation changes if it is necessary to estimate the probability of the simultaneous completion of the work of two developers.
Probability of that event is

$$F(t) = F_1(t)F_2(t) \qquad (26)$$

Substituting F_1 and F_2 from (24) and (25) into the (26) we can have

$$F(t) = \int_0^t f_1(\vartheta)d\vartheta * \int_0^t f_2(\vartheta)d\vartheta \qquad (27)$$

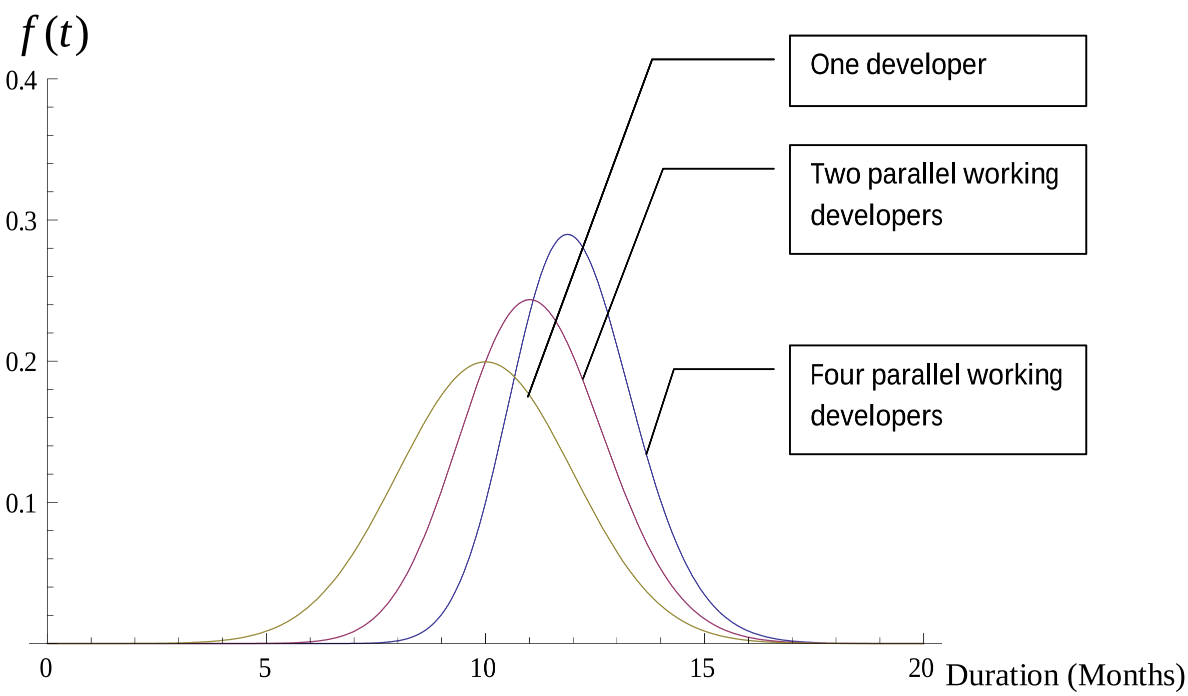

Fig.9 Probability density function to complete the work

Probability density function $f(t)$ of the simultaneous completion of work can be found as the derivative of $F(t)$

$$f(t) = \frac{dF(t)}{dt} = f_1(t)\int_0^t f_2(\vartheta)d\vartheta + f_2(t)\int_0^t f_1(\vartheta)d\vartheta \, . \qquad (28)$$

This density function has a peak at the point for which $\frac{df(t)}{dt} = 0$.

Fig.9 presents the probability density function for the work completion by one person assuming that the completion time has a normal distribution. This figure also contains the densities for two and four parallel working developers with normal distributions of the work completion time for each person.

The results show that the assessment of the simultaneous work of people has its nuances that must be accounted in project timing and reliability assessment.

Conclusions

1. Classical theory of reliability is applicable for the project's reliability analysis.
2. Reliability block diagrams of projects represent itself a mixed system of the serial-parallel human actions.
3. Each human individual or group action has its reliability depending on the difficulty of the task under development.
4. Combining the work flow with the reliability of elements in the form of the serial and parallel human actions it is possible to assess the reliability of the whole project.
5. Each project represents itself a flow of design decisions that can be described with the density function, which is the number of decisions per unit of the project time.
6. Decision density function can be further explored by accounting for the difference between the project decisions by their potential impact on the project risks and reliability.
7. For complex human activities the feasibility analysis of work becoming a task of higher priority, which leads to the emergence of more complex work flows.
8. Structural or high-level design phase of complex projects is a direct consequence of the reliability and risk related requirements.
9. The difficulties of tasks have a significant impact on the serial connection of human activities and must be accounted for the work flow reliability and risk analysis.
10. The duration of the parallel work is a function of the number of parallel human actions which also must be accounted for the work flow reliability and risk analysis.

Future work

- Application of Richard's differential equation for the quantitative presentation of various project staffing profiles.

- Establishment of functional relationships between project goals and the forms of project staffing profiles.

- High-level synthesis of the project staffing profiles based on the goals and objectives of projects, including multi-objective optimization related issues.

- Study of the Rayleigh curve from the point of view of high-level analysis and synthesis of the project's staffing profile, focusing an attention on the inability of this curve to adequately reflect the effort distribution at the starting points of projects.

References

1. Pavel Barseghyan (2009) "Human Effort Dynamics and Schedule Risk Analysis". *PM World Today* – March 2009 (Vol XI, Issue III).
2. Technological Forecasting for Decision Making (Mcgraw Hill Engineering and Technology Management Series) by Joseph Paul Martino. 1992.
3. Growth and Diffusion Phenomena: Mathematical Frameworks and Applications (Texts in Applied Mathematics) by Robert B. Banks. Springer-Verlag. 1993.
4. Weinberg, G. M. Destroying Communication and Control in Software Development CrossTalk, The journal of Defense Software Engineering, April, 2003: http://www.stsc.hill.af.mil/crosstalk/2003/04/Weinberg.html
5. Pavel Barseghyan (2009) "Problems of the Mathematical Theory of Human Work (Principles of mathematical modeling in project management)". *PM World Today* – August 2009 (Vol XI, Issue VIII).
6. Pavel Barseghyan. (2009). Principles of Top-Down Quantitative Analysis of Projects. Part 1: State Equation of Projects and Project Change Analysis. PM World Today – May 2009 (Vol XI, Issue V).
7. Gelfand, I.M. and Fomin, S.V.: Calculus of Variations, Dover Publ., 2000.
8. Pavel Barseghyan. (2009). Task Assignment as a Crucial Factor for Project Success (Probabilistic Analysis of Task Assignment). PM World Today – April 2009 (Vol XI, Issue IV).

About the Author

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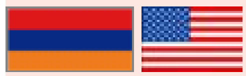

Dr. Pavel Barseghyan is the Vice President of Research for Numetrics Management Systems, has over 40 years experience in academia, the electronics industry, the EDA industry and Project Management Research. Also he is the founder of Systemic PM, LLC, a project management company. Prior to joining Numetrics, Dr. Barseghyan worked as an R&D manager at Infinite Technology Corp. in Texas. He was also a founder and the president of an EDA start-up company, DAN Technologies, Ltd. that focused on high-level chip design planning and RTL structural floor planning technologies. Before joining ITC, Dr. Barseghyan was head of the Electronic Design and CAD department at the State Engineering University of Armenia, focusing on development of the Theory of Massively Interconnected Systems and its applications to electronic design. During the period of 1975-1990, he was also a member of the University Educational Policy Commission for Electronic Design and CAD Direction in the Higher Education Ministry of the former USSR. Earlier in his career he was a senior researcher in Yerevan Research and Development Institute of Mathematical Machines (Armenia). He is an author of nine monographs and textbooks and more than 100 scientific articles in the area of electronic design and EDA methodologies, and tools development. More than 10 Ph.D. degrees have been awarded under his supervision. Dr. Barseghyan holds an MS in Electrical Engineering (1967) and Ph.D. (1972) and Doctor of Technical Sciences (1990) in Computer Engineering from Yerevan Polytechnic Institute (Armenia). Pavel can be contacted at pavel@systemicpm.com.