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The Dynamics of Human Social Behavior: Qualitative Mathematical Description of Human Interactions

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Abstract

Periodicity of processes and their stability for any level of the motion of matter are inseparably linked with each other. From this perspective, the process of interaction between human beings is no exception.

Under the constancy or stability of relations between people are hiding subtle mechanisms of periodic or wave-like behavior. This circumstance is crucial for the analysis of stability of human groups. In particular, it can serve as a conceptual basis for the qualitative mathematical modeling of the human interaction processes. Also such an approach is important from the standpoint of the quantitative study of modes of conflicts between people, their competition and leadership.

Interactions between people are the only mechanism that can help us understand the nuances of their activities, as well as ways to maintain their group stability. It is also a key for understanding performance and risk related problems of design teams.

In this paper are considered qualitative mathematical models of the dynamics of the human group behavior in the form of the system of nonlinear differential equations whose solutions for sustainable human relations are periodic functions of time.

As a basis of mathematical models are serving the concepts of mutual concessions and pressures of people, their profits and losses.

The work consists of several parts. The first part deals with the qualitative mathematical models of interaction between two people in terms of their mutual benefits, concessions and losses.

Keywords: Mathematical models of human interactions, predator – prey model, differential equations of human relations, dynamics of human benefits and losses, human nonlinear behavior, nonlinear dynamics.

Introduction

Spatial and temporal periodicities are underlying the stable forms of the existence of multiple forms of matter. The most prominent examples of such forms are the atoms and the planetary system, the crystal structure of solids and the spatial structure of DNA, and many others. This applies also to interpersonal relations, in the chaos of which, upon close examination, one can find strict periodicities, which lie at the basis of the stable human relations.

From this perspective stable families and successful working groups of people represent a set of strict periodicities, while the unstable human entities tend to have an aperiodic behavior. In turn, the conflicts in the human groups are also a manifestation of their aperiodic behavior.

The same can be said about the global political and economic crises, which also can be characterized as manifestations of the aperiodicities of underlying processes. In quantitative terms, this means that for the early detection and diagnosis of these crises can be used their deviations from the stable periodicities. It is understood that a stable crisis-free regime can be characterized by a set of various periodicities deviations from that and can serve as indicators of impending instability.

This means that the first priority of such a diagnosis is the decomposition of the processes as a sum of stable periodic and aperiodic components.

Interactions between individuals: modes of pressure and concession

Interactions between people are the only mechanism that can help us understand the nuances of their activities, as well as ways to maintain their group stability. This effect between individuals should be represented in the form of two components: the negative impacts and positive impacts. In other words, the impact between people should be structured as a concession and as a means of mutual pressure. Here concession and pressure are understood in their generic sense, including all sorts of moral, psychological, physical, financial and other components.

Consider two individuals who are subject to exchange values and goods. The exchange process on both sides is based on the principles of maximum profit and (or) minimum loss. Each of the parties to obtain benefits resorted to all sorts of pressures and concessions on the other side. For each of the parties, there are two extreme modes of obtaining the values and goods from the other side.

The first extreme is when we give all values and goods the other side without any resistance. In the second mode, the extreme opposite side did not give any values and goods to its first side.
In the first case happens unlimited accumulation of property and goods by the one of the parties on the principle of “growth leads to further growth”.

In the second mode, a gradual disappearance of values and goods is occurred, as in this case, the principle of exhaustion leads to a further decline (in the continued absence of any resources for existence).

This may occur for the reason that each side is ready to indefinitely increase the pressure on the other side, if he or she puts up no resistance. A similar way, each party may accumulate some generic value in the form of capital or in the form of moral values (e.g. in the form of moral satisfaction or simple human pleasure).

Mathematical model that reflect similar mechanisms of pressure of one person to another for both opposing parties leads to the equation of free growth with respect to the generalized value V

$$\frac{dV}{dt} = aV \qquad (1)$$

Also this equation is the reflection of the principle of “growth leads to new growth”.

In other words, this equation reflects the principle of maximum profit at the expense of the other side. In this equation coefficient a represents the rate of growth of benefits, or values. This growth coefficient depends on the properties of people and the nature of the processes under study.

Thus, equation (1) reflects the desire of each party indefinitely accumulate value and benefits at the expense of the other side, of course, if the opposing party puts up no resistance. In the case of two persons involved in the process, this behavior can be reflected by the following equations.

$$\frac{dV_{12}}{dt} = a_{12}V_{12} \qquad (2\text{-}1),$$

$$\frac{dV_{21}}{dt} = a_{21}V_{21} \qquad (2\text{-}2)$$

Here V_{12} and V_{21} are the mutual accumulated values of the person 1 and person 2 with corresponding growth coefficients a_{12} and a_{21}.

In the same way we can obtain an equation for the second case, when the principle of exhaustion leads to a further decline. In this case, the process of reducing the generalized value V can be described by the following differential equation.

$$\frac{dV}{dt} = -bV \qquad (3)$$

Here b is a positive coefficient, reflecting the rate of reduction of the function V. This coefficient along with other factors also depends on the aggressiveness and pliability of people. The meaning of the obtained equation (3) is that if one side loses a permanent source of wealth and property, then available to him accumulated value is rapidly depleted.

In the specific case of two people the similar behavior can be described by the following two equations

$$\frac{dV_{12}}{dt} = -b_{12}V_{12} \qquad (4\text{-}1)$$

$$\frac{dV_{21}}{dt} = -b_{21}V_{21} \qquad (4\text{-}2)$$

Thus the equations (1) and (3) describe the two extreme cases of accumulation and loss of property and wealth.

By applying the first equation for the first person and the second equation for the second person and vice versa, we obtain the following two symmetric systems of differential equations for describing the behavior of a two person group of people.

$$\left.\begin{aligned} \frac{dV_{12}}{dt} &= a_{12}V_{12} \qquad (5\text{-}1) \\ \frac{dV_{21}}{dt} &= -b_{21}V_{21} \qquad (5\text{-}2) \end{aligned}\right\} \qquad (5)$$

$$\left.\begin{aligned}\frac{dV_{12}}{dt} &= -b_{12}V_{12} \qquad (6\text{-}1)\\ \frac{dV_{21}}{dt} &= a_{21}V_{21} \qquad (6\text{-}2)\end{aligned}\right\} \qquad (6)$$

Naturally, between these two extreme modes of mutual pressure and concessions, there are numerous other regimes, which reflect varying degrees of mutual pressure and concessions between the two persons.

The mathematical description of such regimes can be done by improving systems of equations (5) and (6). Systems of these equations describe the dynamics of unilateral gains and losses on both sides, so they are incomplete and cannot describe the balanced and harmonious relationships between people.

This improvement can be done, if we consider that the above extreme regimes are unbalanced, and therefore people will always seek to get out of this situation, moving to a more balanced state.

In reality, if the interests of the people do not match, they will resist the actions of another party. Quantitatively, the resistance of human actions on the other party can be achieved through changes in the coefficients a and b in the systems of equations (5) and (6). But since these equations are symmetrical reflections of the same behavior, then we continue the study of two equations of system (5) separately.

The resistance of the second person to a unilateral increase in the benefits of the first person can reduce the rate of growth a_{12} in the amount of Δa_{12}. As a result the equation (5-1) takes the following form.

$$\frac{dV_{12}}{dt} = (a_{12} - \Delta a_{12})V_{12} \qquad (9)$$

It is clear that the value of Δa_{12} will directly depend on the magnitude of the pressure V_{21}, and with increasing of V_{21} the value of the change Δa_{12} will be increasing too. This means that in the linear approximation can be taken.

$$\Delta a_{12} = d_{21}V_{21} \qquad (10)$$

Here d_{21} is a proportionality coefficient.

Substituting expression (10) into the equation (9), we can obtain

$$\frac{dV_{12}}{dt} = (a_{12} - d_{21}V_{21})V_{12} \qquad (11)$$

Now let's analyze the equation (5-2). Continuous decline of the benefits of the second person will increase its resistance against the first person. As a result coefficient "$-b_{21}$" in the equation (5-2) will increase its value by Δb_{21} (for maintaining the balance between two parties). For this reason, equation (5-2) takes the form.

$$\frac{dV_{21}}{dt} = (\Delta b_{21} - b_{21})V_{21} \qquad (12)$$

As in the previous case, the magnitude of Δb_{21} in the linear approximation is proportional to the benefits or pressure V_{12}.

$$\Delta b_{21} = k_{12}V_{12} \qquad (13)$$

Substituting the expression (13) into the equation (12), we can obtain

$$\frac{dV_{21}}{dt} = (k_{12}V_{12} - b_{21})V_{21} \qquad (14)$$

Combining equations (11) and (14) in one system, we obtain the mathematical model of the behavioral dynamics of two interacting people different goals.

$$\left.\begin{aligned} \frac{dV_{12}}{dt} &= (a_{12} - d_{21}V_{21})V_{12} \qquad (15\text{-}1) \\ \frac{dV_{21}}{dt} &= (k_{12}V_{12} - b_{21})V_{21} \qquad (15\text{-}2) \end{aligned}\right\} \qquad (15)$$

To solve these equations we need the initial values of V_{12} and V_{21} at the initial moment of time $V_{12}(0) = V_{120}$ and $V_{21}(0) = V_{210}$.

It should be emphasized that during the derivation of these equations we do not impose any restriction on the concepts of pressure, value or benefit, understanding this to mean any value and benefits of moral, financial, and other characters.

On the other hand during the derivation of the equations were made some assumptions, which resulted in their specific mathematical form. More comprehensive and detailed analysis could lead to a somewhat different mathematical result, but it would not change the qualitative picture of the investigated phenomena and processes.

The system of equations (15) in the first approximation describes the dynamics of the gains and losses of the interacting two people.

These equations can represent an infinite number of modes of interactions between people, including the daily working relations, negotiations between people, the ways of physical combat, and so on. The system of equations contains a number of converging and diverging, periodic and aperiodic solutions in a position to reflect the variety of situations in human relations.

Furthermore, regardless of the fact that the derivation of the equations is done for the case of two people, the logic of the derivation and the final results are acceptable to a broader class of phenomena and processes, including the groups of people, the competing companies, or states, and so on. The nature of the studied objects specifically does not affect the qualitative picture of the behavioral dynamics. It's no coincidence that these equations by its structure coincide with the known equations of the “predator – prey” model [1].

Simply, the model of “predator – prey” is historically one of the first models, which mathematically gives a qualitative picture of the coexistence of two interacting parties with different goals.

Qualitative mathematical representation of an arbitrary system with conflicting components could lead to the similar equations.

Of course, the specifics of each particular problem will add to this picture some details, but qualitatively the result will be the same.

The equilibrium state of the system of two interacting people

The system, consisting of two persons, may be in equilibrium, if their mutual values and benefits remain constant in time. This means that the derivatives of these quantities in the regime of equilibrium must be equal to zero.

$$\frac{dV_{12}}{dt} = 0 \text{ and } \frac{dV_{21}}{dt} = 0 \qquad (16)$$

Given these conditions, the system of differential equations (15) transforms into the following system of algebraic equations for the gains or benefits in the equilibrium state

$$\left.\begin{aligned} (a_{12} - d_{21}V_{21}^0)V_{12}^0 = 0 \\ (k_{12}V_{12}^0 - b_{21})V_{21}^0 = 0 \end{aligned}\right\} \qquad (17)$$

As a result for the equilibrium state of the system we can have

$$V_{12}^0 = \frac{b_{21}}{k_{12}} \text{ and } V_{21}^0 = \frac{a_{12}}{d_{21}} \;. \qquad (18)$$

Solutions of the equations and their interpretation

The derived equations from the mathematical point of view have been investigated in detail, and are known as a Lotka – Volterra model of “predator - prey” [1].

For this reason let’s move to the discussion and interpretation of some known solutions of these equations in terms of interpersonal relationships. Volterra showed that the general solution of the system of equations (15) reduces to the following expression.

$$\left(\frac{Exp\left[\frac{V_{12}}{V_{12}^0}\right]}{\left[\frac{V_{12}}{V_{12}^0}\right]} \right)^b \left(\frac{Exp\left[\frac{V_{21}}{V_{21}^0}\right]}{\left[\frac{V_{21}}{V_{21}^0}\right]} \right)^a = R \qquad (19)$$

Here, the value of R is determined by the initial values of $V_{12}(0) = V_{120}$ and $V_{21}(0) = V_{210}$.

Graphically the solutions of equations can be presented in two ways - as a function of time and with the aid of the phase plane [1]. Solutions for the variables $V_{12}(t)$ and $V_{21}(t)$ as functions of time are presented in Fig.1.

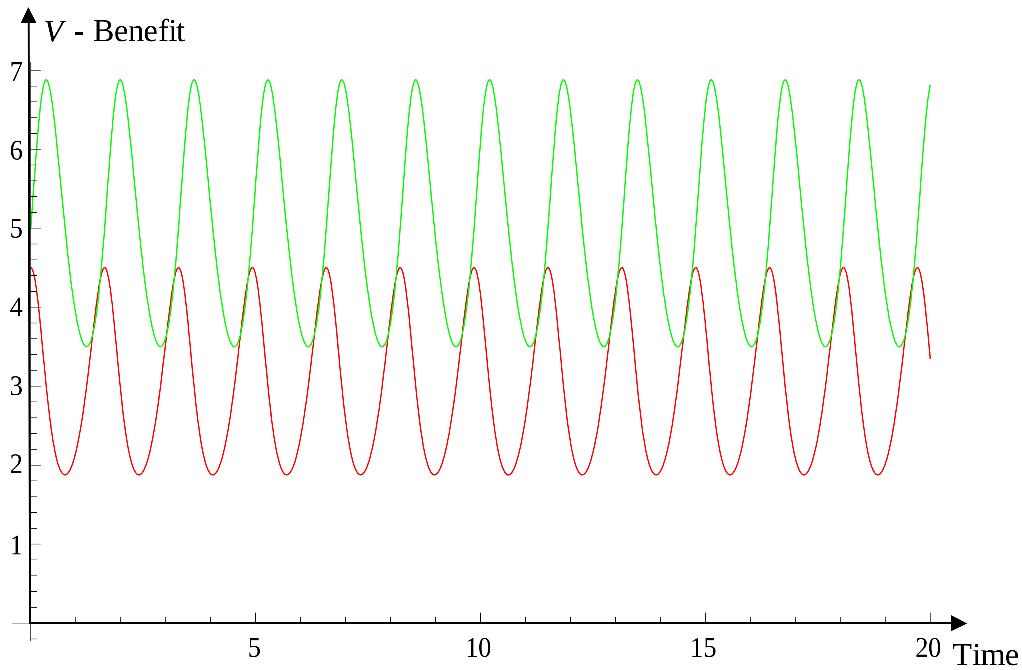

Fig.1 In the stationary mode acquisition and loss of people are periodic functions of time

This figure presents the state of the dynamic equilibrium of relations of the two people with conflicting interests. For the explanation of the behavior of these solutions it is convenient to interpret them as the mutual pressure between people. According to this interpretation, each of the persons seeks to maximize its own profit by increasing the generalized pressure on the second person. This process begins when one party increases the pressure on the other. The second person after some delay is increasing its counter - pressure, as a result forcing the first party is to make certain concessions. As a result, the pressure of the first party on the second side decreases, resulting partial satisfaction of its requirements, or the improvement of its conditions. For this reason, the second side with a certain delay weakens its grip on the first side.

But the first party cannot continue its concessions to the second party infinitely, but at the same time the second party continues its attempts to gain more benefits.

In this situation, the only and an easiest way to maintain the stability of the system is the reduction of the concessions of the first party and its transition to the regime of pressure. As a result, the described cycle is repeated, and so on.

For a detailed analysis of periodic functions, shown in Figure 1, we represent part of that figure in an enlarged form in Fig. 2. In this picture the red curve presents the first side and the green curve - the second side.

A detailed qualitative explanation of the behavior of these periodic curves may well be based on an analysis of equations (1) and (3). The equation (1) describes the process of exponential growth, and the equation (3) - the process of exponential decrease. The first of these trends is presented in equation (15-1), and the second - in the equation (15-2). Other terms of these equations are compensating in one case, the process of growth, and in another case - the process of reduction. As a result, for some values of the coefficients of equations the oscillatory processes occur.

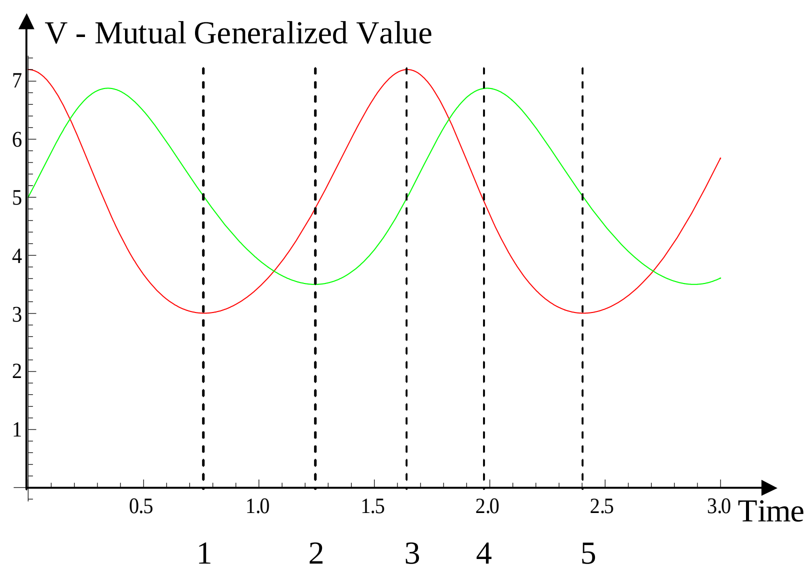

Fig.2 The dynamics of mutual pressures and concessions (or gains and losses) in the stationary mode

Let's start the interpretation of the curves with the conditional point "1", where the first party is in the state of maximum concessions, and by that reason the second party continues to weaken its pressure on the first side. As a result of this reduction of the resistance of the second side, the first party goes from the forced regime of concessions to the regime of pressure, which is expressed in the growth of the red curve from the point of "1". As a consequence, the second side slows down the process of concession or retreat, and, starting from the point "2" under the pressure of the first side, goes to the regime of pressure too.

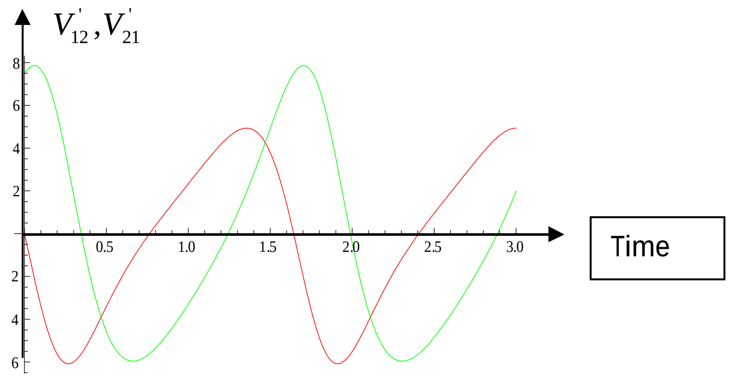

Fig.3 The dynamics of the gains and losses of parties as derivatives of their mutual pressures

Consequently, the increasing pressure of the first party slows down and at the point “3” passes into the regime of concessions, the rate of which reaches its maximum at the point ‘4”.

This is followed by the transition of the second party into the regime of concessions, as it already has forced the first party to make the maximum concessions. The rate of concessions of the second party reaches its maximum at the point of "5", where the patience of the first party ends turning back into the regime of pressure and the accumulation of profit.

The growth rate of benefits and losses of the parties are presented in Fig. 3.

The dynamics of interpersonal relations in the phase plane

As mentioned above, the equation of interpersonal relations can be graphically represented on the phase plane [1]. Fig.4 shows the oscillations of human relationships around the equilibrium point, which is a stable center. The closed curves shown in Fig.4 are the graphic representations of expression (19) for different values of R, with larger fluctuations correspond to the higher values of R. One oscillation period of time means one full rotation on the one of closed curves.

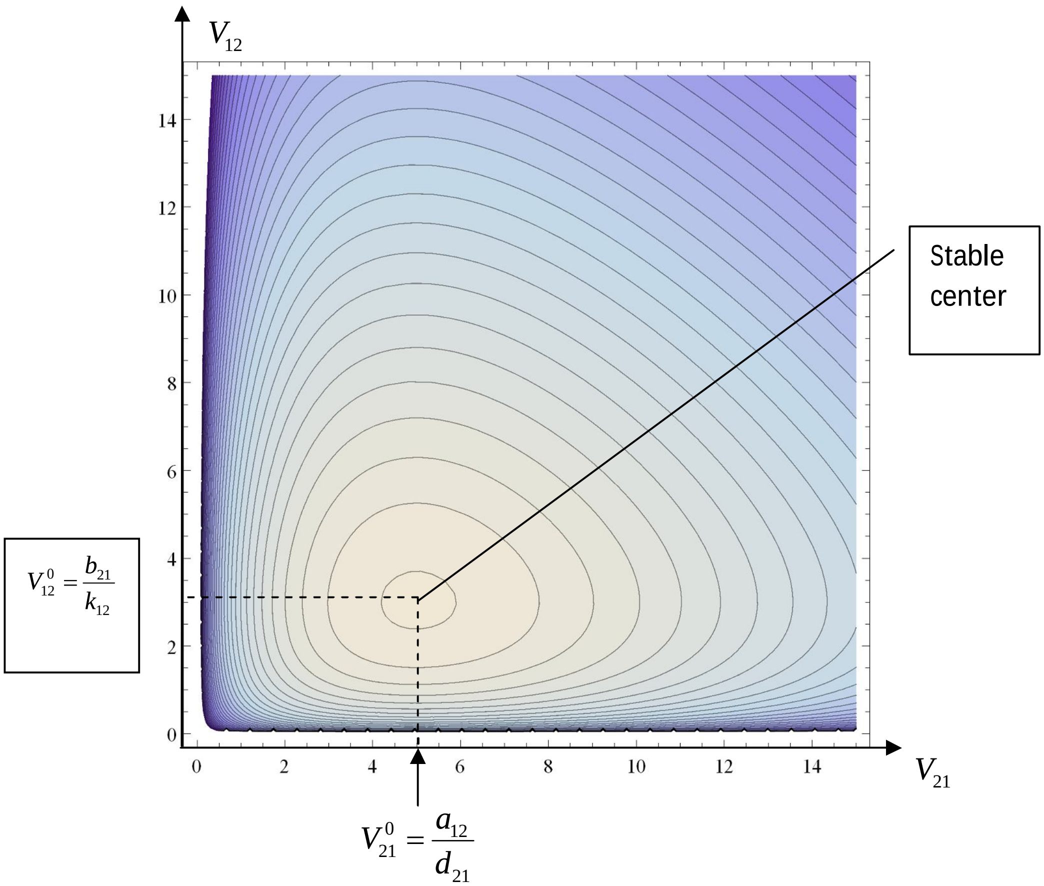

Fig.4 The dynamics of interpersonal relations in the phase plane

Other equivalent forms of equations of the relations between two people

The main difficulty working with differential equations of human relations is that the measurement problems of the generalized values of mutual pressures of people.

For this reason, it is convenient to replace these equations with such equations, whose parameters are relatively easy to measure. From this point of view it is convenient to operate directly with the equations for the values of benefits and losses of the parties. If we denote the people benefits by B_{12} and B_{21}, and the losses through the L_{12} and L_{21}, it is easy to see that the system of equations (15) contains equations for the quantities B_{12} and B_{21}. In the new notations, these equations will have the following form.

$$\left.\begin{aligned}\frac{dB_{12}}{dt} &= (a_{12} - d_{21}B_{21})B_{12} \qquad (25\text{-}1)\\ \frac{dB_{21}}{dt} &= (k_{12}B_{12} - b_{21})B_{21} \qquad (25\text{-}2)\end{aligned}\right\} \qquad (25)$$

Given the fact that in a qualitative sense $B = \frac{\mu}{L}$, where μ - is the coefficient of the inverse proportionality, we can obtain three new graphical presentations of the solutions of the system of equations (15), correspondingly for the following pairs of variables:
$B_{12}(t)$ and $L_{21}(t)$, $L_{12}(t)$ and $B_{21}(t)$, $L_{12}(t)$ and $L_{21}(t)$.

1. For the pair of variables $B_{12}(t)$ and $L_{21}(t)$ the graphical view of the solutions is presented in Fig.7.

These graphs present the dynamics of benefits of the first party and the dynamics of losses for the second party as functions of time.

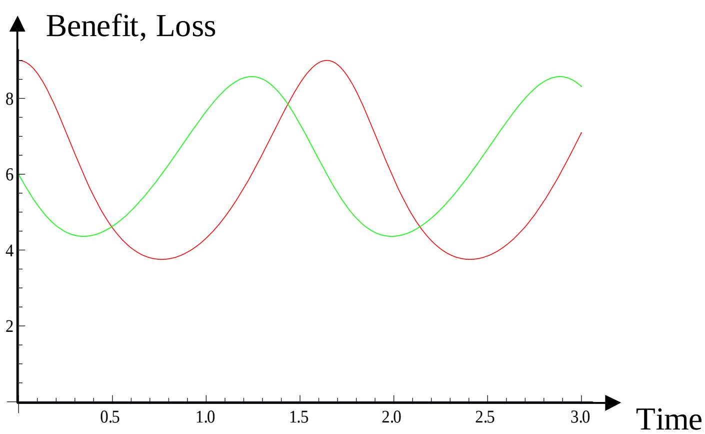

Fig.7 The dynamics of benefits and losses for human relations

Similarly to the previous case, in this case too, the interpretation and explanation of human behavior is based on the causal analysis of their gains and losses.

2. For the pair of variables $L_{12}(t)$ and $B_{21}(t)$ the graphical view of solutions is presented in Fig.8.

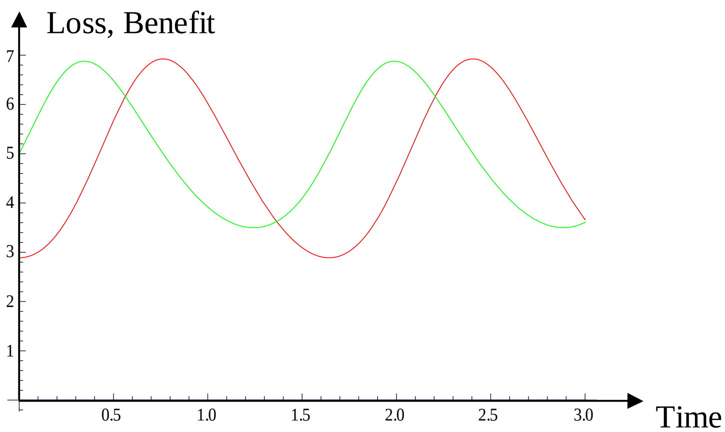

Fig.8 The dynamics of losses and benefits for human relations

3. For the pair of variables $L_{12}(t)$ and $L_{21}(t)$ the graphical view of solutions is presented in Fig.9.

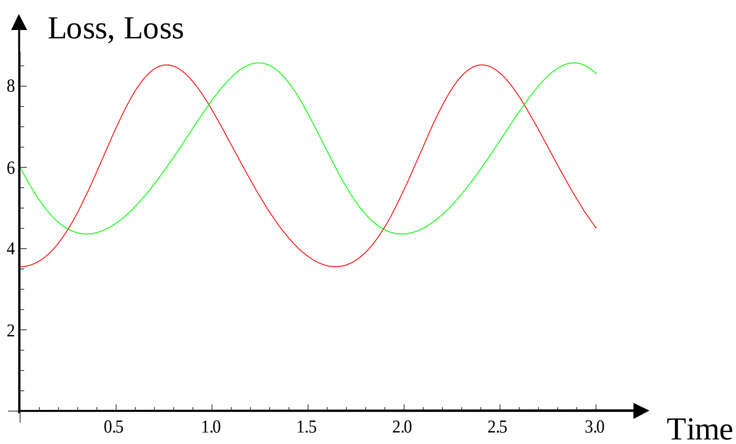

Fig.9 The dynamics of losses for two people

The diversity of the solutions of differential equations of human relations

The solutions of differential equations and their graphical representations mainly reflect the dynamics of relations between the two people in their equilibrium. In other words, these solutions describe the monotonous side of human life – the monotonous family life, the repetitive work, and so on. In mathematical terms, this monotony means periodicity of solutions. But apart from these solutions equations contain an infinite number of other non-periodic solutions that are able to reflect an infinite variety of human relations.

These are different forms of love and hatred, different forms of managing people, including their belief, blackmail, intimidation, bribery, and others. Some of these solutions are already contained in the equations in the form of specific solutions. To capture the other solutions we need to add some new members to the derived equations, or in some cases - to derive new equations. This is already part of the future work on the dynamics of human relations.

Future work

This work can be extended in various directions, including the study of the stability of the behavior of human groups. One of the priorities in this direction is to continue to study the interactions between two people using methods of nonlinear dynamics, including the development of non-stationary models of their behavior.

Models of two interacting people also need to be developed for the case of their self-modulation. This happens with multifaceted interaction of people, when there is interaction between different aspects of their activities. Mathematically, this is the case when the constant coefficients of models have functional relationships with each other. In more general cases, these coefficients can be functions of time too.

Further, the models developed for the case of two people, can serve as a basis for modeling the behavior of large groups of people. In turn, these new models can serve as a basis for further study of conflicts and chaos in human groups.

Conclusions

- The dynamics of human interactions can be presented with differential equations of nonlinear dynamics,

- The derived differential equations for describing the relations between two people in the form coincide with the known model of a predator – prey,

- This means that for further studies of the dynamics of human relations can be used the rich experience, accumulated in the field of mathematical biology,

- Studies show that the periodic solutions of the derived differential equations of human relations reflect the monotonous side of human life, such as monotonous family life, or monotonous relations in the workplace,

- Situations of conflict in human relations quantitatively can be described by the aperiodic solutions of the equations,
- The approach developed in this work and which is based on the methods of nonlinear dynamics, opens new ways for qualitative analysis of the stability of human groups.

References

[1] - J.D. Murray, *Mathematical Biology*. Springer-Verlag, 3rd ed. in 2 vols.: *Mathematical Biology: I. An Introduction*, 2002.

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 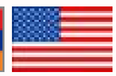

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